

Feb 20, 2020

$$\boxed{A} \boxed{\Pi} = \boxed{Q} \boxed{R} \quad Q^T Q = I$$

Pivoted QR

① Choose column t with largest 2-norm

② Pivot: Swap $A(:, 1)$ and $A(:, t)$

③ $q_1 = A(:, 1) / \|A(:, 1)\|_2$

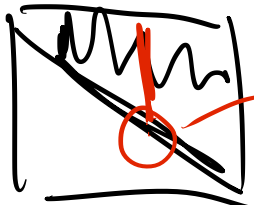
④ $A \leftarrow (I - q_1 q_1^T) A = A - q_1 q_1^T A$

⑤ Repeat on $A(:, 2:n)$

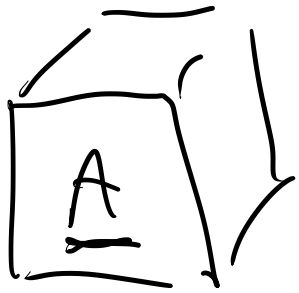
$= R(1, :)$

$$|r_{1,j}| \leq |r_{1,1}|$$

$$|r_{2,2}| \leq |r_{1,1}|$$

R  ≈ 0

Last time: tensors



low-rank approx

$$\min_{\underline{B}} \|\underline{A} - \underline{B}\|_F^2 \quad \text{s.t. } \underline{B} \text{ has rank-} r$$

$$\underline{B} = \sum_{s=1}^r x_s \otimes y_s \otimes z_s \quad \text{Problem: can be ill-posed}$$

Other problems:

(1) no nesting of best orthog. low rank approx

Matrix: $\min_B \|A - B\| \quad \text{s.t. } B \text{ rank } r$

$$B_r = U_r \Sigma_r V_r^T = \sum_{s=1}^r \sigma_s u_s \otimes v_s$$

$$B_{r+1} = B_r + \sigma_{r+1} u_{r+1} \otimes v_{r+1} \quad (\text{Kolda 03})$$

cannot have this for tensors

