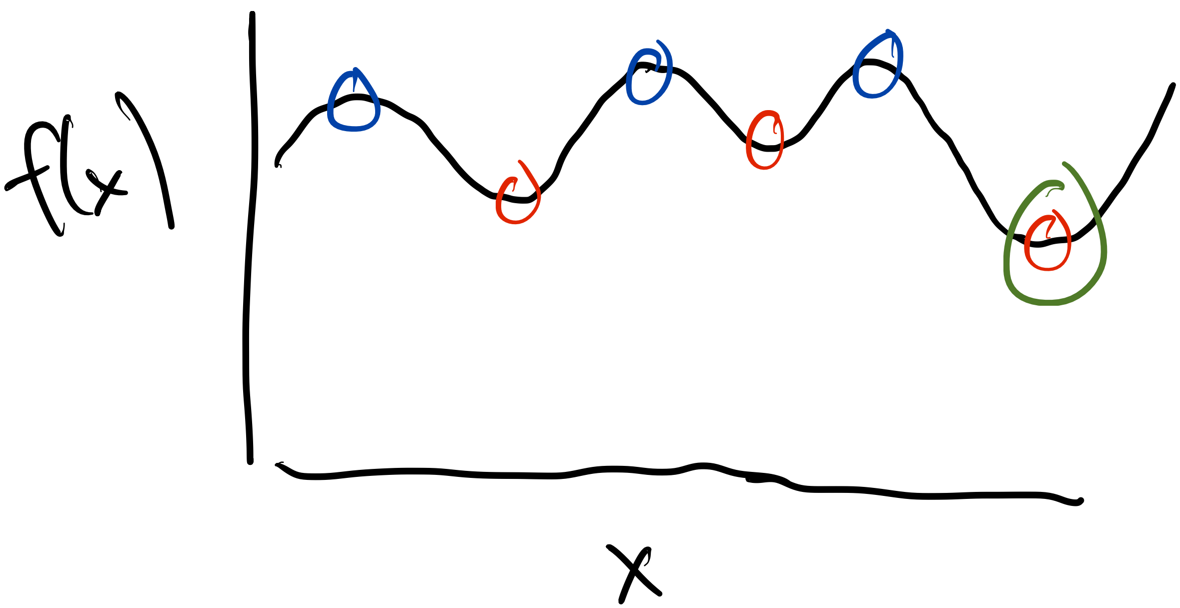
Jan. 23, 2020

$$\min f(x)$$
$$\text{s.t. } x \in \Omega$$ (struck out)

$f: \mathbb{R}^n \to \mathbb{R}$

$\Omega = \mathbb{R}^n$



local mins

global min

convex $\Rightarrow$ local mins are global mins

How do we know if at a local min? (f smooth)

Necessary: $\nabla f(x^*) = 0$

max or a min?

Curvature with Hessian

$H(x) =$ "Hessian at x" $\quad [H(x)]_{ij} = \frac{\partial^2}{\partial x_i \partial x_j} f(x)$

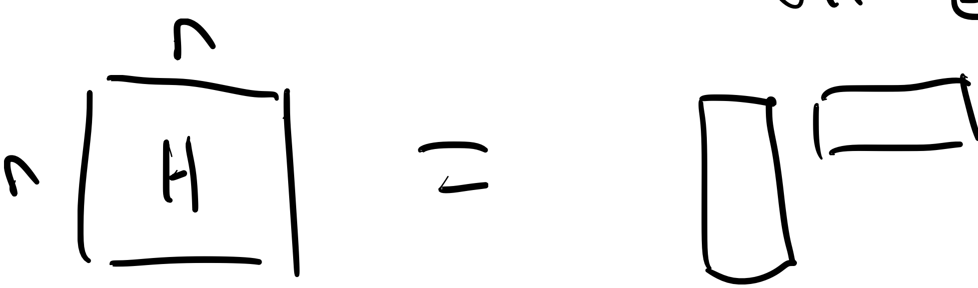
Suppose H is continuous near x^*

<u>Sufficient</u> condition for local min

(1) $\nabla f(x^*) = 0$ + (2) $H(x^*)$ positive def.

(2) $y^T H y > 0 \quad \forall y \in \mathbb{R}^n, \; y \neq 0$

Claim: H is symm. pos. def. (SPD) iff all e-vals > 0



$y^T H y = y^T X^T X y$
$= \|Xy\|_2^2$

Linear least squares

Model: $b \approx a^T x + z$

outcome (arrow to b); intercept (arrow to z)

Error:

$$\begin{matrix} a_1^T & 1 \\ \vdots & \vdots \\ a_n^T & 1 \end{matrix}\Bigg]$$

(1) seem reasonable

$$r = A\hat{x} - b$$

$$\|A\hat{x} - b\|_2^2 = \|A(\hat{x} - x) + r\|_2^2$$

$$\geq \underbrace{\|r\|_2^2}_{\text{bias}} + \underbrace{\|A(\hat{x} - x)\|_2^2}_{\text{variance}}$$

$$\hat{x} = A_{tr}^{+}(b_{tr} + e), \quad x = A_{tr}^{+}(b_{tr} + r_{tr})$$

$$\|A(\hat{x} - x)\|_2 = \|A A_{tr}^{+}(e - r_{tr})\|_2$$

$$\leq \|A\| \, \|A_{tr}^{+}\| \left(\|e\| + \|r_{tr}\|\right)$$