CS 624: Numerical Solution of Differential Equations
Spring 2002
Prelim 1

Handed out: Tues., Feb. 19.

This exam has five questions for a total of 75 points. You have 75 minutes to answer all questions. No books or notes are allowed. Write your answers in the booklet.

1. [**10 points**] Consider solving $u'(t) = u(t)/2$, $u(0) = 1$. Suppose we want to solve this with AB1, AB2, or AB3, such that $u(1)$ is approximated to within 10^{-6}. Roughly what stepsize is necessary for each of AB1, AB2, and AB3 to satisfy this accuracy criterion? Briefly explain your answers.

2. [**20 points**] Consider the second-order IVP $u''(t) = -au(t)$, where a is a positive real scalar. Note that the exact solution of this problem is $u(t) = c_1 \sin(at) + c_2 \cos(at)$. Develop a condition involving k (stepsize), a and the coefficients (α_j's and β_j's) of the LMS method that guarantee stable solution of this problem. Determine in particular the restrictions on k in terms of a necessary for stable solution of this ODE with the EM and BE. Analyze the EM case in more detail, because this is an interesting boundary case of stability. [Hint: First, convert the IVP to first order using the standard trick.]

3. [**15 points**] Consider the two-step LMS formula given by $v^{n+2} = v^{n+1} + k(\beta_1 f^{n+1} + \beta_0 f^n)$. Determine all values of (β_0, β_1) such that (1) the method is consistent, (2) the method is second order, (3) the method is D-stable. For your information, the definition of C_l that determines order is

$$C_l = \sum_{j=0}^{s} j^l \alpha_j - l \sum_{j=0}^{s} j^{l-1} \beta_j$$

where the second term is absent when $l = 0$.

4. [**15 points**] Consider applying BE to the ODE $u'(t) = e^{u(t)}$. Show that on each time step, the nonlinear equation to be solved for BE has either two roots or no roots (or one root with multiplicity 2). Show that if k is sufficiently small, then there are two roots. Which root is the correct value of v^{n+1}? [Note: It is OK to give a nonrigorous answer by, for example, plotting $x - ke^x$ as a function of x and then arguing based on the graph. Point out the correct root using your graph. As for determining the correct root, observe that $v^{n+1} - v^n \to 0$ as $k \to 0$.]

5. [**15 points**] Consider solving $u'(t) = f(u, t)$ using a LMS method with a fixed k, and suppose the solver must determine values of $u(t)$ for all t (not only multiples of k). One approach is for the solver to output v^n as the approximate value for $u(t)$ for all $t \in [kn, k(n + 1)]$. Is this a reasonable approach? [Note: your answer may depend on the order of the LMS method.]