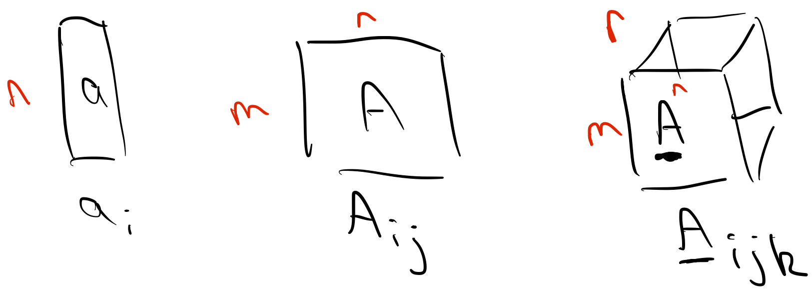
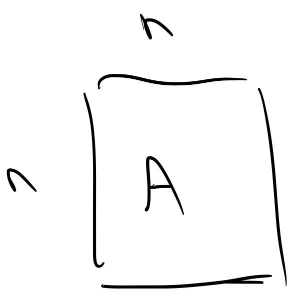
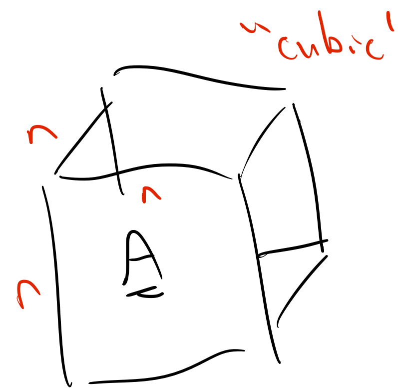
Dec 14, 2020 (GVL 12.4-12.5 4/e)

Tensors / hypermatrices are generalizations of matrices



$\underline{A}_{ijk}$ = info j of patient i at time k





$A_{ij} = \begin{cases} 1 & \text{if } i,j \text{ contact} \\ 0 & \text{o/w} \end{cases}$

$\underline{A}_{ijk} = \begin{cases} 1 & ijk \text{ contact} \\ 0 & \text{o/w} \end{cases}$

Rank and low-rank approx

SVD $\Rightarrow$ best low-rank approx

$$B_k = \underset{B}{\arg\min} \|A - B\|_F^2 \quad \text{s.t. } \operatorname{rank}(B) = k$$

$$A = U\Sigma V^T = \sum_{i=1}^{n} \sigma_i u_i v_i^T$$

$$B_k = \sum_{i=1}^{k} \sigma_i u_i v_i^T$$

$$\operatorname{rank}(B) = \min r \quad \text{s.t. } B = \sum_{s=1}^{r} x_s y_s^T$$

$$B_{ij} = \sum_{s=1}^{r} (x_s)_i (y_s)_j$$

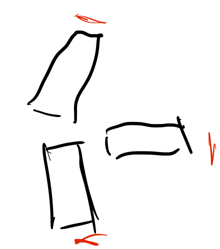
$$B = \underset{n}{\overset{r}{\boxed{X}}}\ \underset{r}{\overset{n}{\boxed{Y^T}}}$$

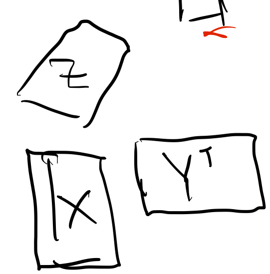
$$\underline{B}_k = \underset{\underline{B}}{\arg\min} \|\underline{A} - \underline{B}\|_F^2 \quad \text{s.t. } \operatorname{rank}(\underline{B}) = k$$

$$\|\underline{M}\|_F^2 = \sum_{ijk} |\underline{M}_{ijk}|^2$$

$$\operatorname{rank}(\underline{B}) = \min r \quad \text{s.t. } \underline{B} = \sum_{s=1}^{r} x_s \otimes y_s \otimes z_s$$

$$\underline{B}_{ijk} = \sum_{s=1}^{r} (x_s)_i (y_s)_j (z_s)_k$$

$\underline{B} \approx \sum$ 

$=$ 

- computing rank($\underline{T}$) is NP-hard

- (matrix)

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