Dec 11, 2020

HW7 due Wed Dec 16

OH: Austin Mon 2:45–3:45pm
John Tues 10–11am

Final:
- 5–7 questions
- some coding
- work on your own
- open everything (cite sources)

Released: Wed Dec 16

Due: Mon Dec 21 5pm ET (CMS)

Iterative methods for eigenproblems

$$\underset{x}{\text{minimize}} \quad \text{error}(Ax, b) \qquad \text{CG: } \|Ax-b\|_{A^{-1}} \Leftrightarrow \|x - A^{-1}b\|_A \quad Ax-b \perp \mathcal{K}_k$$
$$\text{s.t.} \quad x \in \mathcal{K}_k(A, b) \qquad \text{MINRES/GMRES/LSMR} \quad \|Ax-b\|_2$$

$$\underset{x,\,\lambda}{\text{minimize}} \quad \text{error}(A, x, \lambda)$$
$$\text{s.t.} \quad x \in \mathcal{K}_k(A, b)$$

b starting/seed vector

Why might $\mathcal{K}_k(A, b) = \text{span}\{b, \ldots, A^{k-1}b\}$

1. power method: $x_{k-1} = A^{k-1}b / \|A^{k-1}b\| \in \mathcal{K}_k$
2. CG/MINRES/GMRES: error analysis: $\min_{q_k \in P_k} \max_{\lambda_i} |q_k(\lambda_i)|^2 \quad q(0) = 1$
3. $\mathcal{K}_k(A - \sigma I, b) = \mathcal{K}_k(A, b)$
4. $Q_k^T A Q_k = T_k / H_k$ first step: $U^T A U = T/H$
5.

$0 =$

$\hat{v} = V_k y \quad \hat{u} = U_k z$

$0 = U_k^T A V_k y - \hat{\sigma} z$

$B_k y = \hat{\sigma} z$