$\beta_{2k+1} \to \beta_k \quad \beta_{2k} \to \alpha_k$

$A \cdot (A^TA)^{k-1} A^T b = (AA^T)^k b \in K_{k+1}(AA^T, b)$

$AV_k = U_{k+1} \bar{B}_k \quad \bar{B}_k = \begin{pmatrix} \alpha_1 & & \\ \beta_1 & \ddots & \\ & \beta_{k-1} & \alpha_k \\ & & \beta_k \end{pmatrix} \to Av_k = \alpha_k u_k + \beta_k u_{k+1} \in K_{k+1}(AA^T, b)$

$A^T U_{k+1} = V_{k+1} B_{k+1}^T \quad B_{k+1}^T = \begin{pmatrix} \alpha_1 & \beta_1 & & \\ & \ddots & \ddots & \\ & & & \beta_k \\ & & & \alpha_{k+1} \end{pmatrix} \to A^T u_{k+1} = \beta_k v_k + \alpha_{k+1} v_{k+1} \in K_{k+1}(A^TA, A^Tb)$

$A^T (AA^T)^k b$

$= (A^TA)^k A^T b \in K_{k+1}(A^TA, A^Tb)$

$\beta_0 b = u_1 \in K_1(AA^T, b)$

$\alpha_1 v_1 = A^T u_1 \in K_1(A^TA, A^Tb)$

- V_k is an orthobasis for $K_k(A^TA, A^Tb)$ ✓
  U_k $\qquad K_k(AA^T, b)$ ✓

Krylov bases

$AQ_k = Q_{k+1} \bar{H}_k$

$AQ_k = Q_{k+1} \bar{T}_k$

$Q^T A Q = H =$

$\begin{pmatrix} 0 & A \\ A^T & 0 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix} \Sigma$

$$\| A x - b \|$$