

Nov 2, 2020

Problem with repeated evals

$$A(\varepsilon) = \begin{pmatrix} \lambda & \varepsilon \\ 0 & \lambda \end{pmatrix}$$

$$\begin{aligned} p_\varepsilon(z) &= \det(A(\varepsilon) - zI) = (\lambda - z)^2 - \varepsilon \\ &= z^2 - 2\lambda z + \lambda^2 - \varepsilon \\ &= (z - (\lambda + \sqrt{\varepsilon})) (z - (\lambda - \sqrt{\varepsilon})) \end{aligned}$$

evals $\approx \lambda \pm \sqrt{\varepsilon}$ continuous func of ε
but not diff

$$\varepsilon \rightarrow 0 \quad A(0) = \begin{pmatrix} \lambda & \varepsilon \\ 0 & \lambda \end{pmatrix} \quad \text{Jordan block}$$

$O(\varepsilon)$ perturbation $\Rightarrow \sqrt{\varepsilon}$ change cond number $= \infty$

$$s \left\{ \begin{pmatrix} \lambda & \varepsilon & \dots & \varepsilon \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon & \dots & \lambda & \varepsilon \end{pmatrix} \right. \quad \text{evals} = z + O(\varepsilon^{1/s})$$

Better perturbation theory for unsym. matrices

$$Ax = \lambda x, \lambda \text{ simple, } \|x\|_2 = 1$$

$$S^{-1}AS = \text{diag}(J_1, \dots, J_k) \quad J_i = \begin{cases} \left(\begin{array}{ccc} \lambda_i & & 0 \\ & \ddots & \\ 0 & & \lambda_i \end{array} \right) \end{cases} \quad J_i e_1 = \lambda_i \underline{e_1}$$

$$AS = S \text{diag}(J_1, \dots, J_k) \quad e_s^T J_i = \lambda_i \underline{e_s^T} \quad e_s^T e_1 = 0$$

$$S^{-1}A = \text{diag}(J_1, \dots, J_k) S^{-1}, \quad \|y\|_2 = 1$$

$S^{-1}AS = \Lambda \Rightarrow$ right evecs given by cols of S
left evecs given by rows of S^{-1}

$$(A + \delta A)(x + \delta x) = (\lambda + \delta \lambda)(x + \delta x)$$

$$Ax = \lambda x$$

$$(A\delta x + \delta Ax) = \lambda \delta x + \delta \lambda x + \text{higher-order terms}$$

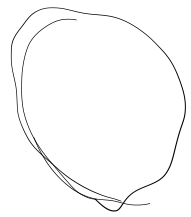
+ higher-order

$$|\delta \lambda| \leq \|\delta A\|_2 + O(\|\delta A\|^2)$$

(2) \rightarrow Jordan block (nontrivial)
 $\rightarrow \infty$



$$\|x^{-1}Ex\|_2$$



E symm

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