Nov 2, 2020

Problem with repeated evals

$$A(\varepsilon) = \begin{pmatrix} \lambda & 1 \\ \varepsilon & \lambda \end{pmatrix}$$

$$p_\varepsilon(z) = \det(A(\varepsilon) - zI) = (\lambda - z)^2 - \varepsilon$$
$$= z^2 - 2\lambda z + \lambda^2 - \varepsilon$$
$$= \left(z - (\lambda + \sqrt{\varepsilon})\right)\left(z - (\lambda - \sqrt{\varepsilon})\right)$$

evals $= \lambda \pm \sqrt{\varepsilon}$ continuous func of ε
but not diff

$\varepsilon \to 0$ $A(0) = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$ Jordan block

$O(\varepsilon)$ perturbation $\Rightarrow$ $\sqrt{\varepsilon}$ change cond number $= \infty$

$$s\left\{\begin{pmatrix} \lambda & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ \varepsilon & & & \lambda \end{pmatrix}\right.$$ evals $= z + O(\varepsilon^{1/s})$

Better perturbation theory for unsymm. matrices

$Ax = \lambda x$, λ simple, $\|x\|_2 = 1$

$S^{-1}AS = \mathrm{diag}(J_1, \dots, J_k)$ $\quad J_i = s\left\{\begin{pmatrix} \lambda_i & 1 & & 0 \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ 0 & & & \lambda_i \end{pmatrix}\right.$ $\quad J_i e_1 = \lambda_i e_1$

$AS = S\,\mathrm{diag}(J_1, \dots, J_k)$ $\quad e_s^T J_i = \lambda_i e_s^T \quad e_s^T e_1 = 0$

$S^{-1}A = \mathrm{diag}(J_1, \dots, J_k)\, S^{-1}$

$, \quad \|y\|_2 = 1$

$S^{-1}AS = \Lambda \Rightarrow$ right eves given by cols of S
left rows S^{-1}

$(A + \delta A)(x + \delta x) = (\lambda + \delta\lambda)(x + \delta x)$

$Ax = \lambda x$

$(A\delta x + \delta A x) = \lambda \delta x + \delta\lambda x$ + higher-order terms

$|\delta\lambda| \leq \|\delta A\|_2 + O(\|\delta A\|^2)$

+ higher-order

(2) → Jordan block (nontrivial)
→ ∞

$\|X^{-1}EX\|_\infty$

Esymm

=