Oct 21, 2020

Power method

$z = A x_k$

$x_{k+1} = z / \|z_k\|_2$

$\tilde{\lambda}_{k+1} = x_{k+1}^H A x_{k+1}$

$\lambda_1 \approx \lambda_2, \ |\lambda_2| > |\lambda_3| > \ldots \geq |\lambda_n|$

PM: slow! $|\lambda_1 / \lambda_2| \approx 1$

Inverse iteration

σ close to λ_1

$\Rightarrow$ close to λ_2

Idea: look for subspace V corresponding to λ_1, λ_2

$V = E_{\lambda_1} \cup E_{\lambda_2}$

V invariant subspace

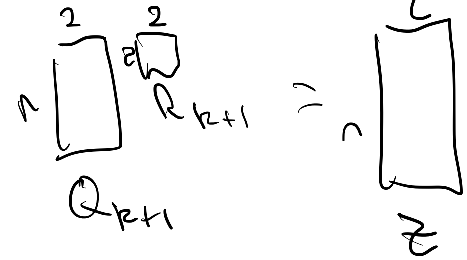
$AV \subseteq V$

Problem: need basis for V

Idea: $Q_0 \ n \times 2 \quad Q_0^H Q_0 = I$

$z = A Q_k$

$Q_{k+1} R_{k+1} = Z$



More generally... orthogonal iteration

Let $1 \leq p < n$ $\quad Q_0^H Q_0 = I, \ Q_0 \ n \times p$

$z = A Q_k$

$Q_{k+1} R_{k+1} = Q_k$

$p = 1$ is just the PM

Evals?

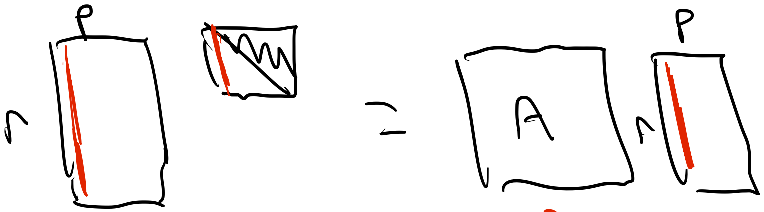
$\tilde{\Lambda} = \operatorname{diag}(Q_{k+1}^H A Q_{k+1})$

if q column of Q_{k+1}, $Aq \approx \lambda q$

$q^H A q \approx \lambda$

eigensolver$(Q_{k+1}^H A Q_{k+1})$ $\quad p \times p$

$Q_{k+1} R_{k+1} = A Q_k$



$q_{k+1,1} r_{11} = A \, q_{k,1}$

$Q_{k+1}(:, 1:j)$

$:, 1:j)$

(sketch)

Reduced / thin
econ QR

$Q_{k+1} R_{k+1} = Z_k$

$\text{span}(Q_{k+1}) = \text{span}(Z_k)$

$Q_k R_k = A \cdot Q_{k-1} \quad = \text{span}(A^2 Q_{k-1})$

$\vdots$

$(\lambda_p I)^k$