Oct 19, 2020 Want $x \neq 0$, λ $Ax = \lambda x$ or $\{(x_i, \lambda_i)\}$

Power iteration

$$y_{k+1} = A x_k$$
$$x_{k+1} = y_{k+1} / \|y_{k+1}\|_2$$
$$\tilde{\lambda}_{k+1} = x_{k+1}^H A x_{k+1}$$

Assume $A = S \Lambda S^{-1}$

$|\lambda_1| > |\lambda_2| \geq \dots \geq |\lambda_n|$

$$x_{k+1} = A^k x_0 / \|A^k x_0\|_2$$
$$A^k = (S\Lambda S^{-1})(S\Lambda S^{-1}) \cdots (S\Lambda S^{-1})$$
$$= S \Lambda^k S^{-1}, \quad x_0 = S z$$
$$A^k x_0 = S\Lambda^k S^{-1} S z = S \begin{pmatrix} \lambda_1^k z_1 \\ \vdots \\ \lambda_n^k z_n \end{pmatrix}$$
$$= \lambda_1^k z_1 \cdot S \begin{pmatrix} 1 \\ (\lambda_2/\lambda_1)^k z_2/z_1 \\ \vdots \\ (\lambda_n/\lambda_1)^k z_n/z_1 \end{pmatrix}$$

$\Rightarrow S e_1$

= first eigenvector

Problems

1. $z_1 = 0$? random x_0
2. $|\lambda_2| = |\lambda_1|$
3. $|\lambda_2/\lambda_1| \approx 1$
4. only one eigenpair

sym: $A = V \Lambda V^T$

(v_1, λ_1)

$(I - v_1 v_1^T) A (I - v_1 v_1^T)$

Example: Markov chains

$P_{ij} = \text{Prob}(X_{t+1} = i \mid X_t = j)$

column stochastic

P "nice" $\Rightarrow$

$\lambda_1 = 1, \quad Pv = v, \quad v \geq 0$

$|\lambda_1| > |\lambda_2|$

v = stationary distribution

$y_i = (Pv)_i = \sum_j P_{ij} v_j$

$= \Pr(X_{t+1} = i \mid X_t = j) \Pr(X_t = j)$

at state i)

X_0 starting distribution

$= \Pr(X_1 = i \mid x_0)$

$\Lambda^k z$

$= S($