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(1) $\underset{n \times n}{A}\,\underset{n \times 1}{x} = \underset{n \times 1}{b}$

(2) $\arg\min_x \| \underset{m \times n}{A}\,\underset{n \times 1}{x} - \underset{m \times 1}{b} \|_2$

$$A^T A \hat{x} = A^T b$$

(3) $Ax = \lambda x \quad x \neq 0 \qquad (A - \lambda I)x = 0$ eigenpair (x, λ) $(\alpha x, \lambda)$

$$p(\lambda) = \det(A - \lambda I) \quad \text{characteristic polynomial}$$

$$= (\lambda_1 - \lambda)(\lambda_2 - \lambda) \cdots (\lambda_n - \lambda)$$

Consequences:

- exactly n eigenvalues (with algebraic multiplicities)
- even if $A \in \mathbb{R}^{n \times n}$, λ_j might be complex $\lambda_j \in \mathbb{C}$
- $A \in \mathbb{R}^{n \times n}$, λ_j is eval, $\bar{\lambda}_j$ is eval
- if $n \geq 5$, can't have exact algorithms

$$\min/\max \quad x^T M x$$
$$\text{s.t.} \quad x^T x = 1$$

$$L(x,\gamma) = x^T M x - \gamma(x^T x - 1)$$
$$\nabla_x L(x,\gamma) = 2Mx - 2\gamma x = 0 \Rightarrow Mx = \gamma x$$
$$\frac{\partial}{\partial \gamma} L(x,\gamma) = x^T x - 1 = 0 \Rightarrow \|x\|_2 = 1$$