

Sep 25, 2020

Gaussian elimination with sparsity

$A$  sparse,  $PA = LU$ , are  $L, U$  also sparse?

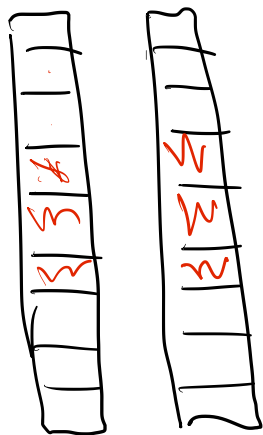
$PAQ = LU$   $P, Q$  permutation

$$Ax = b \Rightarrow \underbrace{PAQ}_{LU} Q^T x = Pb \quad Q^T x = y \quad x = Qy$$

choosing  $Q$  to encourage sparsity in  $L, U$   
"sparse direct"

Compressed sparse column (CSC)

values rows



col ptr



4 column 2  
7 in indices  
4 to 6

$$y = Ax$$
$$y = \sum_j A(:, j) x_j$$
$$O(\underline{nnz}(A))$$

$A$

$$\begin{pmatrix} x & x & x & x & x \\ x & & & & \\ x & & & & \\ x & & & & \\ x & & & & \end{pmatrix}$$

first step:  $L(2:5, 1) = A(2:5, 1) / A(1, 1)$

$$A(2:5, 2:5) = \underline{L(2:5, 1)} \underline{A(1, 2:5)}$$

$$\begin{pmatrix} x & x & x & x & x \\ x & & & & \\ x & & & & \\ x & & & & \\ x & & & & \end{pmatrix}$$

$$L(2:5, 1) = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}$$

$$A(1, 2:5) = (0 \ 0 \ 0 \ x)$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & 0 \end{pmatrix}$$

$$\begin{pmatrix} x & & & & \\ & x & & & \\ & & \dots & & \\ & & & & x \end{pmatrix}$$

L

$$\begin{pmatrix} x & & & & \\ & x & & & \\ & & \dots & & \\ & & & & x \end{pmatrix}$$

U

$$\begin{aligned} & \text{rank}(L) + \text{rank}(U) \\ &= \text{rank}(A) \end{aligned}$$

