Sep 18, 2020

Solving square linear systems of equations

$$\overset{n}{\underset{n}{\boxed{A}}} \cdot \overset{1}{\underset{n}{\boxed{x}}} = \overset{1}{\underset{n}{\boxed{b}}}$$

$\cancel{A^{-1}b}$ $\Rightarrow$ $\cancel{\mathrm{inv}(A)\cdot b}$

just for math notation

bad!

$x:\ Ax=b$

1. destroy structure
2. slower than GE
3. not as stable

We will study a factorization approach:

$$PA = LU$$

perm. matrix

that is <u>backward stable</u>

"Direct" solver

$PA = LU$, A nonsingular

$$Ax = b$$

$$PAx = Pb$$

$$LUx = Pb$$

① $c = Pb$ $O(n)$

② $Ly = c$ forward sub $O(n^2)$

③ $Ux = y$ backward sub $O(n^2)$

$$\begin{array}{ccc} L & y & c \\ \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix} & \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = & \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \end{array}$$

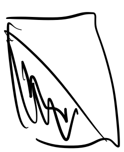
$1 \cdot y_1 = 1 \Rightarrow y_1 = 1$

$2 \cdot y_1 + y_2 = 1 \Rightarrow y_2 = -1$

$3y_1 + 2y_2 + y_3 = 3$

$3 - 2 + y_3 = 3$ $y_3 = 2$

$\square$



```
y = 0
for i = 1:n
    s = c_i
    for j = 1:i-1
        s -= l_ij y_j
    y_i = s / l_ii
```