HW for 2019-10-28  
(due: 2019-11-04)

You may (and should) talk about problems with each other and with me, providing attribution for any good ideas you might get. Your final write-up should be your own.

1: On the border  
Suppose the bordered matrix  
\[ M(s) = \begin{bmatrix} A - sI & b \\ c^T & 0 \end{bmatrix} \]
is nonsingular, and consider the linear system  
\[ \begin{bmatrix} A - sI & b \\ c^T & 0 \end{bmatrix} \begin{bmatrix} f(s) \\ g(s) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \]

- Show that \( g(\lambda) = 0 \) iff \( \lambda \) is an eigenvalue of \( A \).
- Modify the hw7newton code (Julia or MATLAB) to implement the Newton iteration  
  \[ \sigma_{k+1} = \sigma_k - g(\sigma_k)/g'(\sigma_k). \]

  You should see quadratic convergence in the tester, as indicated by the \( g(\sigma_{k+1}) \) having roughly the order of magnitude of \( g(\sigma_k)^2 \).

2: Real rotations  
Suppose \( A \in \mathbb{R}^{n \times n} \) has a unique (algebraic multiplicity 1) complex conjugate pair of eigenvalues \( \mu \exp(\pm i\theta) = \alpha + \beta i \) with maximal modulus \( \mu > |\lambda| \) for all other eigenvalues \( \lambda \) and corresponding eigenvectors \( u \pm vi \). Show that power iteration from a random starting vector in \( \mathbb{R}^n \) gives the sequence  
\[ v_k \approx u \cos(k\theta + \gamma) - v \sin(k\theta + \gamma) \]
for large \( k \).

3: Shifted solver  
Suppose \( H \in \mathbb{R}^{n \times n} \) is given upper Hessenberg matrix. Write a QR-based solver that runs in \( O(n^2) \) time to solve linear systems of the form \( (H - \sigma I)x = b \). Your code should satisfy the interface in the class repository.