Approximate letter grade intervals. A = [90,100], A- = [80-89], B = [70-79], C = [60-69]

1. Complete the following function so that it performs as specified:

   ```matlab
   function [x,y] = HessSolver(H,b)
   % H is an n-by-n nonsingular matrix with H(i,j) = 0 whenever i>j+1.
   % b is a column n-vector.
   % x solves Hx = b and y solves H'*y = b.
   [L,U,P] = lu(H');
   y = U\(L\(P*b)); %Lw = Pb, Uy = w
   x = P*(L'
   %7 points
   [L,U,P] = lu(H');
   %9 points
   y = U\(L\(P*b)); %Lw = Pb, Uy = w
   %9 points
   x = P*(L'

2. Complete the following MATLAB function so that it performs as specified:

   ```matlab
   function [x,y] = Solver(A,b,c)
   % A is n-by-n, symmetric, and positive definite.
   % b is n-by-1 and c is (n-1)-by-1
   % x satisfies Ax = b and y satisfies A(1:n-1,1:n-1)y = c
   % Use \ to solve triangular systems and assume the availability of a function G = chol(A) that returns the Cholesky factor of a symmetric positive definite matrix.
   % Since
   % \[ A = \begin{bmatrix} A_1 \\ v^T \\ \alpha \end{bmatrix} = \begin{bmatrix} G_1 & 0 \\ w^T & \mu \end{bmatrix} \begin{bmatrix} G_1 & 0 \\ w^T & \mu \end{bmatrix}^T = GG^T \]
   % it follows that G(1:n-1,1:n-1) is the Cholesky factor of A(1:n-1,1:n-1). (13 points) Thus
   
   [n,n] = size(A);
   G = chol(A);
   % 12 points
   x = G\(G\b);
   % 13 points
   y = G(1:n-1,1:n-1)\(G(1:n-1,1:n-1)\c);
   ```
3. Suppose \( A \in \mathbb{R}^{n \times n} \) is nonsingular that \( A = U\Sigma V^T \) is its SVD.

Facts:
\[
A = \sum_{i=1}^{n} \sigma_i u_i v_i^T, \quad A^{-1} = (U\Sigma V^T)^{-1} = V\Sigma^{-1}U^T = \sum_{i=1}^{n} \sigma_i u_i v_i^T / \sigma_i
\]

and so
\[
x = \sum_{i=1}^{n} \frac{u_i^T b}{\sigma_i} v_i
\]

(a) Give an example of a vector \( b \in \mathbb{R}^n \) such that if \( Ax = b \) then \( \| x \|_2 = \| b \|_2 / \sigma_{\min} \) where \( \sigma_{\min} \) is the smallest singular value of \( A \).

If \( b = u_n \) then \( x = v_n / \sigma_n \) has the required property. (12 points)

(b) Give an example of a matrix \( E \in \mathbb{R}^{n \times n} \) so that \( \kappa_2(A+E) = 2\kappa_2(A) \). Here, \( \kappa(\cdot) \) is the 2-norm condition. (13 points)

let
\[
E = -(\sigma_n/2)u_n v_n^T
\]

4. Throughout this problem you may use \( \backslash \) to solve triangular linear systems. Assume that \( S, T \in \mathbb{R}^{n \times n} \) are upper triangular and \( B \in \mathbb{R}^{n \times n} \). We want to solve the linear system \( SX - XT = B \) for \( X \in \mathbb{R}^{n \times n} \). Assume that the system is nonsingular.

(a) How would you compute \( X(:,1) \)? (5 points)

Compare first columns: \( SX(:,1) - T(1,1)X(:,1) = B(:,1) \) and so
\[
X(:,1) = (S - T(1,1)\text{eye}(n,n))\backslash B(:,1);
\]

(b) How would you compute \( X(:,2) \) given that you have computed \( X(:,1) \)? (5 points)

Compare second columns: \( SX(:,2) - T(1,2)X(:,1) - T(2,2)X(:,2) = B(:,2) \) and so
\[
X(:,2) = (S - T(2,2)\text{eye}(n,n))\backslash (B(:,2) + T(1,2)X(:,1));
\]

(c) Complete the following MATLAB function so that it performs as specified: (15 points)

```matlab
function X = Sylvester(S,T,B)
% S and T are n-by-n upper triangular matrices.
% B is n-by-n
% X is n-by-n and satisfies SX - XT = B

[n,n] = size(S);
X = zeros(n,1);
for j=1:n
    d = B(:,j);
    for i=1:j-1
        d = d + T(i,j)*X(:,i);
    end
    X(:,j) = (S - T(j,j)*eye(n,n))\d;
end
```