**CS 621: Assignment 1**

Due: Friday, September 20, 2002 (In Lecture)

Scoring on each problem is on a 0-1-2-3 scale: 3 = complete success, 2 = overlooked a small point, 1 = some of the right idea, 0 = missed the point of the problem. One point will be deducted for insufficiently commented code. Test drivers and related material will be posted on the course website. For each problem submit output and a listing of all the scripts/functions that you had to write/modify in order to produce the output. You are allowed to discuss background issues with other students, but the codes you submit must be your own.

**Problem 1.** Recall Horner’s nested multiplication scheme that is normally used for polynomial evaluation. To evaluate \( p(x) = c_1 + c_2x + \cdots + c_dx^{d-1} \) at \( x = z \) we proceed as follows:

\[
pval = c(d) \\
\text{for } k = d-1:-1:1 \\
\quad pval = z*pval + c(k)
\]

end

Develop an efficient, fully vectorized saxpy implementation for the following MATLAB function

\[
\text{function } y = \text{PolyVec}(c, A, p)
\]

% c is a column n-vector
% A is an n-by-n matrix with lower bandwidth p. Assume mpcon
% y is the first column of \( c(1)I + c(2)A + c(3)A^2 + \cdots + c(n)A^{n-1} \)

Use the Horner idea and exploit A’s band structure. You may find the discussion in §11.2.4 helpful. Test your code on the script P1.m that is available on the website. Submit output and a listing of PolyVec.

**Problem 2.** Suppose \( Q \in \mathbb{R}^{n \times n} \) is orthogonal and \( v \in \mathbb{R}^{n} \). Consider the matrix

\[
M = [ v \ | \ Qv \ | \ Q^2v \ | \ \cdots \ | \ Q^{n-1}v ]
\]

Think about the structure of the matrix \( A = M^T M \). Implement the following MATLAB function so that it performs as specified:

\[
\text{function } y = \text{KrylovProd}(Q, v, x)
\]

% Q is an n-by-n orthogonal matrix.
% v and x are column n-vectors
% y = M^x * M \ where \ M = [ v \ | \ Qv \ | \ Q^2v \ | \ \cdots \ | \ Q^{n-1}v ]

Test your code on the script P2.m that is available on the website. Submit output and a listing of KrylovProd.

**Problem 3.** For given 2-by-2 real matrix \( A \) define the set \( S_p(A) = \{ Av : \| v \|_p = 1 \} \). Write a MATLAB function \( \text{pNormImage} (A, p) \) that opens up a new figure window and plots \( S_p(A) \). It should include the command \( \text{axis equal} \) so that the \( x \) and \( y \) axes have the same scale. The plot should be based on a couple of hundred points. Test your code on the script P3.m that is available on the website. Submit output and a listing of pNormImage. To do this problem you will have to generate a couple of hundred points around the set of all unit \( p \)-norm vectors. Apply \( A \) to each of these and then “connect the dots” in the range space. Your code should be vectorized and efficient.

**Problem 4.** Define the matrices \( W_2, W_4, W_8, \ldots \) as follows

\[
W_{2m} = \begin{cases} 
\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} & \text{if } m = 1 \\
\begin{bmatrix} W_m & W_m \\ W_m & -W_m \end{bmatrix} & \text{if } m > 1 
\end{cases}
\]

Complete the following MATLAB function so that it performs as specified:
function y = FastProd(x)
% x is a column n-vector and n is a power of 2.
% y = \mathbb{W}_n(n) * x

Hint. Look at the product

\[
\begin{bmatrix}
\mathbb{W}_m & \mathbb{W}_m \\
\mathbb{W}_m & -\mathbb{W}_m
\end{bmatrix}
\begin{bmatrix}
y \\
z
\end{bmatrix}
\]

where \( y, z \in \mathbb{R}^n \). Your implementation should be recursive and involve \( O(n \log n) \) flops. Take a look at the Strassen algorithm §1.3.7 for guidance on the design of a recursive matrix procedure. Test your code on the script P4.m that is available on the website. Submit output and a listing of FastProd.