CS 621: Matrix Computations
Fall 2001
Prelim 1

Handed out: Thurs., Sep. 27.

This is a timed 75-minute closed-book and closed-note exam. Write all answers in the exam booklet. There are a total of 75 points on this exam.

1. [10 points] The following posting appeared yesterday on the internet newsgroup comp.soft-sys.matlab. The author of this posting seems to be a bit confused about stability and conditioning. Please explain what advice you’d give him. Incidentally, RCOND in Matlab means the reciprocal of the condition number of a matrix.

   I get some warnings like RCOND=10e-17 matrix close to singular or badly scaled. The consequence is that the results of my simulation, which uses matrixes of 400 order, are completely wrong. The points I obtain in the plot shouldn’t be greater than ’1’, and they actually are bigger. Is there a possibility of calling another program like mathematica or C, to obtain better results with a similar computing time? In that case is there anybody who has a mex file in C language which solves the inverse matrix without this problem. Thanks in advance

2. [10 points] Explicitly work out the condition number in the $\infty$-norm of

   \[
   \begin{pmatrix}
   1 & 1 \\
   1 & 1 + a
   \end{pmatrix}
   \]

   as a function $a$. [Note: Obtain $A^{-1}$ any way you’d like. One way to obtain $A^{-1}$ from $A$ is to repeatedly solve linear systems of the form $Ax = e_i$ for $x$, where $e_i$ is the $i$th column of the identity matrix.]

3. [15 points] Consider the division operation of two real numbers $a/b$ with $b \neq 0$. Show that this operation is well-conditioned for all data.

4. [15 points] In class we showed that $\|AB\|_\infty \leq \|A\|_\infty \|B\|_\infty$. Find an example of an $A, B \in \mathbb{R}^{2 \times 2}$ such that this inequality becomes an equation. On the other hand, find an example of $A, B \in \mathbb{R}^{2 \times 2}$ such that the right-hand side is 1000 times larger than the left.

5. [10 points] Let $A$ be an $m \times n$ matrix, let $P$ be an $m \times m$ permutation matrix, and let $Q$ be an $n \times n$ permutation matrix. Show that $\|A\|_p = \|PAQ\|_p$ for any matrix $p$-norm.

6. [15 points] Let $A$ be an $n \times n$ matrix that is upper triangular except for the first column and last row, which are both full. In other words, $A(i,j) = 0$ for all $i,j$ satisfying $1 < j < i < n$. How many flops, accurate to the leading term, are required to solve $Ax = b$ using Gaussian elimination without pivoting, followed by forward and back substitution?