```
⊢ ∀[T:Type]. ∀[P:T → ℙ]. ∀[C:ℙ].  ((∀x:T. (C ⇒ (P x))) ⇒ C ⇒ (∀x:T. (P x)))
|
BY RepeatFor 3 ((UD  THENA Auto))
|
[1]. T: Type
[2]. P: T → ℙ
[3]. C: ℙ
⊢ (∀x:T. (C ⇒ (P x))) ⇒ C ⇒ (∀x:T. (P x))
|
BY RepeatFor 2 ((D 0 THENA Auto))
|
4. ∀x:T. (C ⇒ (P x))
5. C
⊢ ∀x:T. (P x)
|
BY (D 0 THENA Auto)
|
6. x: T
⊢ P x
|
BY (SimpleInstHyp ⌜x⌝ 4· THENA Auto)
|
7. C ⇒ (P x)
⊢ P x
|
BY D 7
|\
| ⊢ C
| |
1 BY NthHyp 5
 \
  7. P x
  ⊢ P x
  |
  BY NthHyp 7


Extract:

λf,c,x. (f x c)
  where f : ∀x:T. (C ⇒ (P x))
        c : C
```

```
⊢ ∀[P,Q:ℙ].  ((¬(P ⇒ Q)) ⇒ P ⇒ (¬Q))
|
BY RepeatFor 2 ((UD THENA Auto))
|
[1]. P: ℙ
[2]. Q: ℙ
⊢ (¬(P ⇒ Q)) ⇒ P ⇒ (¬Q)
|
BY RepeatFor 2 ((D 0 THENA Auto))
|
3. ¬(P ⇒ Q)
4. P
⊢ ¬Q
|
BY Unfold `not` 0
|
⊢ Q ⇒ False
|
BY (D 0 THENA Auto)
|
5. Q
⊢ False
|
BY Unfold `not` 3
|
3. (P ⇒ Q) ⇒ False
⊢ False
|
BY D 3
|\
| 3. P
| 4. Q
| ⊢ P ⇒ Q
| |
1 BY (D 0 THENA Auto)
| |
| 5. P
| ⊢ Q
| |
1 BY NthHyp 4
 \
  3. P
  4. Q
  5. False
  ⊢ False
  |
  BY NthHyp 5
```

```
Extract: λf,p1,q. (f (λp2.q))
    where f : ¬(P ⇒ Q) ≡ ((P ⇒ Q) ⇒ False)
          p1 : P
           q : Q
          p2 : P
```

```
⊢ ∀[T:Type]. ∀[A,B:T → ℙ]. {∃x:T. ((A x) ⇒ (B x)) ⟺ (∀x:T. {A x}) ⇒ (∃x:T. (B x))}
|
BY Auto
|
1. T: Type
2. A: T → ℙ
3. B: T → ℙ
⊢ {∃x:T. ((A x) ⇒ (B x)) ⟺ (∀x:T. {A x}) ⇒ (∃x:T. (B x))}
|
BY RepeatFor 4 ((D 0 THENA Auto))
|\
| 4. ∃x:T. ((A x) ⇒ (B x))
| ⊢ {(∀x:T. {A x}) ⇒ (∃x:T. (B x))}
| |
1 BY RepeatFor 2 ((D 0 THENA Auto))
| |
| 5. ∀x:T. {A x}
| ⊢ {∃x:T. (B x)}
| |
1 BY D 4
| |
| 4. x: T
| 5. (A x) ⇒ (B x)
| 6. ∀x:T. {A x}
| ⊢ {∃x:T. (B x)}
| |
1 BY (InstHyp [⌜x⌝] 6· THENA Auto)
| |
| 7. {A x}
| ⊢ {∃x:T. (B x)}
| |
1 BY (ExposeClassical THENA Auto)
| |
| 7. A x
| ⊢ {∃x:T. (B x)}
| |
1 BY (ElimClassical THENA Auto)
| |
| ⊢ ∃x:T. (B x)
| |
1 BY (InstConcl [⌜x⌝]· THENA Auto)
| |
| ⊢ B x
| |
1 BY D 5
| |\
| | 5. ∀x:T. {A x}
| | 6. A x
| | ⊢ A x
| | |
1 2 BY Hypothesis
|  \
|   5. ∀x:T. {A x}
|   6. A x
|   7. B x
|   ⊢ B x
```

```
|   |
1   BY Hypothesis
 \
  4. (∀x:T. {A x}) ⇒ (∃x:T. (B x))
  ⊢ {∃x:T. ((A x) ⇒ (B x))}
  |
  BY (ClassicalContradiction THENA Auto)
  |
  5. ¬(∃x:T. ((A x) ⇒ (B x)))
  ⊢ {∃x:T. ((A x) ⇒ (B x))}
  |
  BY D 4
  |\
  | 4. ¬(∃x:T. ((A x) ⇒ (B x)))
  | ⊢ ∀x:T. {A x}
  | |
  1 BY (D 0 THENA Auto)
  | |
  | 5. x: T
  | ⊢ {A x}
  | |
  1 BY (ClassicalContradiction THENA Auto)
  | |
  | 6. ¬(A x)
  | ⊢ {A x}
  | |
  1 BY D 4
  | |
  | 4. x: T
  | 5. ¬(A x)
  | ⊢ ∃x:T. ((A x) ⇒ (B x))
  | |
  1 BY (InstConcl [⌜x⌝]· THENA Auto)
  | |
  | ⊢ (A x) ⇒ (B x)
  | |
  1 BY (D 0 THENA Auto)
  | |
  | 6. A x
  | ⊢ B x
  | |
  1 BY D 5
  | |
  | 5. A x
  | ⊢ A x
  | |
  1 BY Hypothesis
   \
    4. ¬(∃x:T. ((A x) ⇒ (B x)))
    5. ∃x:T. (B x)
    ⊢ {∃x:T. ((A x) ⇒ (B x))}
    |
    BY D 5
    |
    5. x: T
    6. B x
    ⊢ {∃x:T. ((A x) ⇒ (B x))}
```

```
|
BY (ElimClassical THENA Auto)
|
⊢ ∃x:T. ((A x) ⇒ (B x))
|
BY (InstConcl [⌜x⌝]· THENA Auto)
|
⊢ (A x) ⇒ (B x)
|
BY (D 0 THENA Auto)
|
7. A x
⊢ B x
|
BY Hypothesis
```

```
⊢ ∀[A,B:ℙ].  {(((A ⇒ B) ⇒ A) ⇒ A}
|
BY (D 0 THENA Auto)
|\
| 1. A: ℙ
| ⊢ ∀[B:ℙ]. {(((A ⇒ B) ⇒ A) ⇒ A}
| |
1 BY (D 0 THENA Auto)
| |\
| | 2. B: ℙ
| | ⊢ {(((A ⇒ B) ⇒ A) ⇒ A}
| | |
1 2 BY RepeatFor 2 ((D 0 THENA Auto))
| | |
| | 3. (A ⇒ B) ⇒ A
| | ⊢ {A}
| | |
1 2 BY (ClassicalContradiction THENA Auto)
| | |
| | 4. ¬A
| | ⊢ {A}
| | |
1 2 BY D 3
| | |\
| | | 3. ¬A
| | | ⊢ A ⇒ B
| | | |
1 2 3 BY (D 0 THENA Auto)
| | | |
| | | 4. A
| | | ⊢ B
| | | |
1 2 3 BY D 3
| | | |
| | | 3. A
| | | ⊢ A
| | | |
1 2 3 BY Hypothesis
| |  \
| |   3. ¬A
| |   4. A
| |   ⊢ {A}
| |   |
1 2   BY ElimClassical
| |   |
| |   ⊢ A
| |   |
1 2   BY Hypothesis
|  \
|   2. B: ℙ
|   3. {x:Unit| (((A ⇒ B) ⇒ A) ⇒ A}
|   ⊢ Ax ∈ {x:Unit| (((A ⇒ B) ⇒ A) ⇒ A}
|   |
1   BY Auto
 \
  1. A: ℙ
```

```
2. B: ℙ
3. {x:Unit| ((A ⇒ B) ⇒ A) ⇒ A}
⊢ Ax ∈ {x:Unit| ((A ⇒ B) ⇒ A) ⇒ A}
|
BY Auto
```

```
⊢ ∀[T:Type]. ∀[P:T → ℙ]. ∀[C:ℙ].  (((∃x:T. (P x)) ⇒ C) ⇒ (∀x:T. ((P x) ⇒ C)))
|
BY RepeatFor 3 ((UD THENA Auto))
|
[1]. T: Type
[2]. P: T → ℙ
[3]. C: ℙ
⊢ ((∃x:T. (P x)) ⇒ C) ⇒ (∀x:T. ((P x) ⇒ C))
|
BY (D 0 THENA Auto)
|
4. (∃x:T. (P x)) ⇒ C
⊢ ∀x:T. ((P x) ⇒ C)
|
BY (D 0 THENA Auto)
|
5. x: T
⊢ (P x) ⇒ C
|
BY (D 0 THENA Auto)
|
6. P x
⊢ C
|
BY D 4
|\
| 4. x: T
| 5. P x
| ⊢ ∃x:T. (P x)
| |
1 BY (InstConcl [⌜x⌝]· THENA Auto)
| |
| ⊢ P x
| |
1 BY NthHyp 5
 \
  4. x: T
  5. P x
  6. C
  ⊢ C
  |
  BY NthHyp 6
```

Extract:

```
λf,x,p. (f <x, p>)
  where f : (∃x:T. (P x)) ⇒ C
        p : P x
```