CS6180 Lecture 27 – The Age of Intelligent Machines

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Abstract

This lecture and class discussion will consider ways in which proof assistants will evolve and what the consequences could be for science and for life in the age of intelligent machines. We will also review the foundational value of type theory and pursue observations and questions that naturally arise as we explore these topics and others of general interest.

1 Automated Reasoning

One origin of automated reasoning research is Artificial Intelligence (AI) with the fundamental paper by Newell, Shaw, and Simon *Empirical Explorations with the Logic Theory Machine: A case study in heuristics* [4]. Another regular source of articles was the Machine Intelligence volumes, e.g. *Fixedpoint Induction and Proofs of Program Properties* [5]. This topic automated reasoning topic continues to this day to have strong ties to other research in AI.

Proof assistants such as Agda, Coq, HOL, Nuprl, etc. are designed to help users create completely formal proofs. Recently a key focus of applications has been on using existing tools to build *provably correct software*. There is *immediate economic value* to this activity, and industry is willing to support it. It is assumed that some of the funding for applied work will enhance the effectiveness of proof assistants. To a limited extent this is the case, but it is also clear that there are many elements of automated reasoning research that will enhance every aspect of the use of proof assistants.

As we have noted previously, if a country were to invest heavily in automated reasoning research, it is highly likely and predictable that the results would be less costly development of *provably correct software*. In addition there is the distinct possibility of fundamental advances in the effectiveness of proof assistants in helping solve important problems in mathematics and science. An example of this are the results of Dr. Bickford and his collaborators Coquand and Anders, as they confirmed the *constructive validity* of the Univalence Axiom in homotopy theory. Major improvements in automated reasoning have the potential to be one of the next major milestones in the steady and remarkable advance of computer science. One related area of research with expected very high pay off in increasing the effectiveness of proof assistants is machine learning. We will hear about this later today in the lecture of Scott Aaronson, who is a graduate of Cornell CS.

At Cornell we have direct experience with how AI results have made it possible to solve open problems in mathematics [2, 3, 1] – the last one was open for sixty seven years. I am hoping to see many more examples of this use of proof assistants, something they are well positioned to help us accomplish.

References

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