Consensus, impossibility results and Paxos

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Consensus... a classic problem
- Consensus abstraction underlies many distributed systems and protocols
  - \( N \) processes
  - They start execution with inputs \( \in \{0,1\} \)
  - Asynchronous, reliable network
  - At most 1 process fails by halting (crash)
  - Goal: protocol whereby all “decide” same value \( v \), and \( v \) was an input

Distributed Consensus

Asynchronous networks
- No common clocks or shared notion of time (local ideas of time are fine, but different processes may have very different “clocks”)
- No way to know how long a message will take to get from \( A \) to \( B \)
- Messages are never lost in the network

Quick comparison...

<table>
<thead>
<tr>
<th>Asynchronous model</th>
<th>Real world</th>
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<tbody>
<tr>
<td>Reliable message passing,</td>
<td>just resend until acknowledged;</td>
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<tr>
<td>unbounded delays</td>
<td>often have a delay model</td>
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<tr>
<td>No partitioning faults (“wait until</td>
<td>May have to operate “during”</td>
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<tr>
<td>over”)</td>
<td>partitioning</td>
</tr>
<tr>
<td>No clocks of any kinds</td>
<td>Clocks but limited sync</td>
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<tr>
<td>Crash failures, can’t detect</td>
<td>Usually detect failures with</td>
</tr>
<tr>
<td>reliably</td>
<td>timeout</td>
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Fault-tolerant protocol
- Collect votes from all \( N \) processes
  - At most one is faulty, so if one doesn’t respond, count that vote as 0
  - Compute majority
  - Tell everyone the outcome
  - They “decide” (they accept outcome)
  - … but this has a problem! Why?

Jenkins, if I want another yes-man, I’ll build one!
What makes consensus hard?
- Fundamentally, the issue revolves around membership
  - In an asynchronous environment, we can’t detect failures reliably
  - A faulty process stops sending messages but a “slow” message might confuse us
  - Yet when the vote is nearly a tie, this confusing situation really matters

Fischer, Lynch and Patterson
- A surprising result
  - Impossibility of Asynchronous Distributed Consensus with a Single Faulty Process
  - They prove that no asynchronous algorithm for agreeing on a one-bit value can guarantee that it will terminate in the presence of crash faults
    - And this is true even if no crash actually occurs!
    - Proof constructs infinite non-terminating runs

Core of FLP result
- They start by looking at a system with inputs that are all the same
  - All 0’s must decide 0, all 1’s decides 1
  - Now they explore mixtures of inputs and find some initial set of inputs with an uncertain (“bivalent”) outcome
  - They focus on this bivalent state

Self-Quiz questions
- When is a state “univalent” as opposed to “bivalent”?
- Can the system be in a univalent state if no process has actually decided?
- What “causes” a system to enter a univalent state?

Self-Quiz questions
- Suppose that event e moves us into a univalent state, and e happens at p.
  - Might p decide “immediately”?
- Now sever communications from p to the rest of the system. Both event e and p’s decision are “hidden”
  - Does this matter in the FLP model?
  - Might it matter in real life?

Bivalent state
- System starts in S*
- Events can take it to state S0
- Events can take it to state S1
- S* denotes bivalent state
- S0 denotes a decision 0 state
- S1 denotes a decision 1 state
- Sooner or later all executions decide 0
- Sooner or later all executions decide 1
Bivalent state

System starts in $S_*$

Events can take it to state $S_1$

Events can take it to state $S_0$

$e$ is a critical event that takes us from a bivalent to a univalent state. Eventually we’ll “decide” $e$.

Bivalent state

They delay $e$ and show that there is a situation in which the system will return to a bivalent state.

Bivalent state

In this new state they show that we can deliver $e$ and that now, the new state will still be bivalent!

Bivalent state

Notice that we made the system do some work and yet it ended up back in an "uncertain" state. We can do this again and again.

Core of FLP result in words

- In an initially bivalent state, they look at some execution that would lead to a decision state, say “0”
  - At some step this run switches from bivalent to univalent, when some process receives some message $m$
  - They now explore executions in which $m$ is delayed

Core of FLP result

- Initially in a bivalent state
- Delivery of $m$ would make us univalent but we delay $m$
- They show that if the protocol is fault-tolerant there must be a run that leads to the other univalent state
- And they show that you can deliver $m$ in this run without a decision being made
Core of FLP result
- This proves the result: a bivalent system can be forced to do some work and yet remain in a bivalent state.
  - We can “pump” this to generate indefinite runs that never decide
  - Interesting insight: no failures actually occur (just delays). FLP attacks a fault-tolerant protocol using fault-free runs!

Intuition behind this result?
- Think of a real system trying to agree on something in which process p plays a key role
- But the system is fault-tolerant: if p crashes it adapts and moves on
- Their proof “tricks” the system into treating p as if it had failed, but then lets p resume execution and “rejoin”
  - This takes time... and no real progress occurs

But what did “impossibility” mean?
- In formal proofs, an algorithm is totally correct if
  - It computes the right thing
  - And it always terminates
  - When we say something is possible, we mean “there is a totally correct algorithm” solving the problem

But what did “impossibility” mean?
- FLP proves that any fault-tolerant algorithm solving consensus has runs that never terminate
  - These runs are extremely unlikely (“probability zero”)
  - Yet they imply that we can't find a totally correct solution
  - “Consensus is impossible” thus means “consensus is not always possible”

Solving consensus
- Systems that “solve” consensus often use a membership service
  - This GMS functions as an oracle, a trusted status reporting function
- Then consensus protocol involves a kind of 2-phase protocol that runs over the output of the GMS
  - It is known precisely when such a solution will be able to make progress

GMS in a large system
- Global events are inputs to the GMS
- Output is the official record of events that mattered to the system
Paxos Algorithm

- Distributed consensus algorithm
  - Doesn’t use a GMS… at least in basic version… but isn’t very efficient either
  - Guarantees safety, but not liveness.
- Key Assumptions:
  - Set of processes that run Paxos is known a-priori
  - Processes suffer crash failures
  - All processes have Greek names (but translate as “Fred”, “Cynthia”, “Nancy”…)

Paxos “proposal”

- Node proposes to append some information to a replicated history
- Proposal could be a decision value, hence can solve consensus
- Or could be some other information, such as “Frank’s new salary” or “Position of Air France flight 21”

Paxos Algorithm

- Proposals are associated with a version number.
- Processors vote on each proposal. A proposal approved by a majority will get passed.
  - Size of majority is “well known” because potential membership of system was known a-priori
  - A process considering two proposals approves the one with the larger version number.

Paxos Algorithm

- 3 roles
  - proposer
  - acceptor
  - Learner
- 2 phases
  - Phase 1: prepare request ←→ Response
  - Phase 2: Accept request ←→ Response

Phase 1: (prepare request)

1. A proposer chooses a new proposal version number n, and sends a prepare request (“prepare”,n) to a majority of acceptors:
   - Can I make a proposal with number n?
   - If yes, do you suggest some value for my proposal?

Phase 1: (prepare request)

2. If an acceptor receives a prepare request (“prepare”, n) with n greater than that of any prepare request it has already responded, sends out (“ack”, n, n’, v’) or (“ack”, n, ⊥, ⊥)
   - Responds with a promises not to accept any more proposals numbered less than n.
   - Suggest the value v of the highest-number proposal that it has accepted if any, else ⊥
Phase 2: (accept request)

(3) If the proposer receives responses from a majority of the acceptors, then it can issue an accept request ("accept", n, v) with number n and value v:
   (a) n is the number that appears in the prepare request.
   (b) v is the value of the highest-numbered proposal among the responses.

(4) If the acceptor receives an accept request ("accept", n, v), it accepts the proposal unless it has already responded to a prepare request having a number greater than n.

Learning the decision

- Whenever acceptor accepts a proposal, respond to all learners ("accept", n, v).
- Learner receives ("accept", n, v) from a majority of acceptors, decides v, and sends ("decide", v) to all other learners.
- Learners receive ("decide", v), decide v.

In Well-Behaved Runs

Paxos is safe...

Intuition:
- If a proposal with value v is decided, then every higher-numbered proposal issued by any proposer has value v.

Safety (proof)

- Suppose (n, v) is the earliest proposal that passed. If none, safety holds.
- Let (n', v') be the earliest issued proposal after (n, v) with a different value v' ≠ v.
- As (n', v') passed, it requires a major of acceptors. Thus, some process approve both (n, v) and (n', v')
- As (n', v') passed, it must receive a response ("ack", n', j, v') to its prepare request, with n < j < n'. Consider (j, v) we get the contradiction.
Liveness

- Per FLP, cannot guarantee liveness
- Paper gives us a scenario with 2 proposers, and during the scenario no decision can be made.

Liveness (cont.)

- Omissions cause the Liveness problem.
  - Partitioning failures would look like omissions in Paxos
  - Repeated omissions can delay decisions indefinitely (a scenario like the FLP one)
  - But Paxos doesn’t block in case of a lost message
    - Phase I can start with new rank even if previous attempts never ended

Liveness (cont.)

- As the paper points out, selecting a distinguished proposer will solve the problem.
  - “Leader election”
  - This is how the view management protocol of virtual synchrony systems works... GMS view management “implements” Paxos with leader election.
  - Protocol becomes a 2-phase commit with a 3-phase commit when leader fails

A small puzzle

- How does Paxos scale?
  - Assume that as we add nodes, each node behaves iid to the other nodes
  - … hence likelihood of concurrent proposals will rise as $O(n)$
  - Core Paxos: 3 linear phases… but expected number of rounds will rise too… get $O(n^2)$… $O(n^3)$ with failures...

How does Paxos scale?

- Another, subtle scaling issue
  - Suppose we are worried about the memory in use to buffer pending decisions and other messages
  - Under heavy load, round trip delay to reach a majority of the servers will limit the “clearing” time
  - Works out to something like an $O(n \log n)$ or $O(n^2)$ cost depending on how you implement the protocol. This is a kind of “time-space” complexity that has never really been studied... we’ll see why it matters in an upcoming lecture

Paxos in real life

- Used but not widely. For example, Google uses Paxos in their lock server
  - One issue is that Paxos gets complex if we need to reconfigure it to change the set of nodes running the protocol
  - Another problem is that other more scalable alternatives are available
Summary

- Consensus is “impossible”
  - But this doesn’t turn out to be a big obstacle
  - We can achieve consensus with probability one in many situations
- Paxos is an example of a consensus protocol, very simple
- We’ll look at other examples Thursday