

# Hella Crunk Opfibrations THE THIRD

## Multiple Fors

for (showing in Showings) <sup>independent of showing!</sup>  
 for (address in Addresses)  
 if (showing.theatre == address.theatre  
 && address.city == Ithaca)  
 showing.movie  
 => Lists all the movies playing in Ithaca

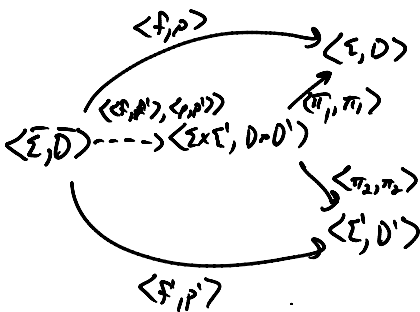
## Adjacent Fors

for (showing, address in Showings, Addresses)  
 ...  
 How to query over two databases?

## Categorical Product of Databases

$$\langle \Sigma, D \rangle \times \langle \Sigma', D' \rangle = \langle \Sigma \times \Sigma', D \times D' \rangle$$

## Categorical Product



## Multiset Databases

Object:  $\langle \Sigma, E, e \rangle$   
 where  $\Sigma$  is a schema  
 and  $E$  is a set  
 and  $e: E \rightarrow \mathcal{B}[\Sigma]$   
 (i.e.  $E$  is the set of entries  
 and  $e$  specifies their values)

Morphisms:  $(f_E, f_e, f_c)$   
 $: \langle \Sigma, E, e \rangle \rightarrow \langle \Sigma', E', e' \rangle$   
 where  
 $f_\Sigma: \Sigma \rightarrow \Sigma'$   
 $f_E: E \rightarrow E'$   
 $f_e: E \xrightarrow{f_E} E' \xrightarrow{f_c} \mathcal{B}[\Sigma']$   
 $e \downarrow \cong \downarrow e'$   
 $[\Sigma] \xrightarrow{f_\Sigma} [\Sigma']$

### Products of Multiset Databases

nt	string	=	nt	string
$\boxed{1}$	$\times$ $\boxed{\text{"hello"}}$	=	$\boxed{1}$	$\boxed{\text{"hello"}}$
$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\times$ $\boxed{\text{"hello"}}$	=	$\begin{bmatrix} 1 & \text{"hello"} \\ 1 & \text{"hello"} \end{bmatrix}$	
$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\times$ $\begin{bmatrix} \text{"hello"} \\ \text{"hello"} \end{bmatrix}$	=	$\begin{bmatrix} 1 & \text{"hello"} \\ 1 & \text{"hello"} \\ 1 & \text{"hello"} \\ 1 & \text{"hello"} \end{bmatrix}$	

for (smaller & string tables) if (string table = all other tables)

$$\langle \Sigma, D \rangle \xrightarrow{\text{the sum}} \langle \Sigma, D' \rangle$$

↓

$$\langle \Sigma, ? \rangle$$

### Fibres of Multiset Databases

$f: \langle \Sigma, G, c \rangle \rightarrow \langle \Sigma', E', c' \rangle$

consider morphisms:  $\langle \Sigma, E, c \rangle \rightarrow \langle \Sigma, G, c \rangle$

such that  $UF = id_{\Sigma}$

or  $|P|$

This category is called the fibre over  $\Sigma$

### Vertical Morphism in Multiset Database over $\Sigma$

$\langle E, f_E \rangle: \langle E, c \rangle \rightarrow \langle E', c' \rangle$

here  $f_E: E \rightarrow E'$

$$\begin{array}{ccc} E & \xrightarrow{f_E} & E' \\ c \downarrow & & \downarrow c' \\ \langle \Sigma \rangle & & \langle \Sigma \rangle \end{array}$$

### Fibred Products for Multiset Databases (over $\Sigma$ )

is called pullback

$$\begin{array}{ccc} E & & E' \\ \downarrow \pi_1 & & \downarrow \pi_2 \\ E \times_{\Sigma} E' & & E' \\ \downarrow e & & \downarrow e' \\ \langle \Sigma \rangle & & \langle \Sigma \rangle \end{array}$$

$E \times_{\Sigma} E' = \{(e, e') \mid e \in E, e' \in E', c(e) = c(e')\}$

Matrix example:  $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

### Database Join

$\langle D_1, D_2, \Sigma \rangle \rightarrow \langle D', E' \rangle$

↓  $\langle D, \Sigma \rangle$

$$\begin{array}{ccc} \Sigma \times E' & \xrightarrow{\quad} & E' \\ \downarrow & & \downarrow \\ \Sigma & \xrightarrow{\quad} & \Sigma \times E' \end{array}$$

or as

```
for (d ∈ D)
```

```
  for (d' ∈ D')
```

```
    if (d.common = d'.common)
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```
      <d.numerator, d.denominator, d'.numerator>
```