Hella Crunk Opfibrations THE THIRD

Multiple Fors

for (showing in Showings)
    for (address in Addresses) ↙ independent of showing!
        if (showing.theatre = address.theatre
            && address.city = Ithaca)
        showing.movie

⇒ Lists all the movies playing in Ithaca

Adjacent Fors

for (showing, address in Showings, Addresses)

...

How to query over two databases?

Categorical Product of Databases

$$\langle \Sigma, D\rangle \times \langle \Sigma', D'\rangle = \langle \Sigma\times\Sigma', D\times D'\rangle$$

Categorical Product

$$\begin{tikzcd}
 & & \langle \Sigma, D\rangle \\
\langle \bar{\Sigma}, \bar{D}\rangle \arrow[urr, bend left, "\langle f,\rho\rangle"] \arrow[drr, bend right, "\langle f',\rho'\rangle"'] \arrow[r, dashed, "\langle\langle f,f'\rangle,\langle\rho,\rho'\rangle\rangle"] & \langle \Sigma\times\Sigma', D\times D'\rangle \arrow[ur, "\langle\pi_1,\pi_1\rangle"'] \arrow[dr, "\langle\pi_2,\pi_2\rangle"] & \\
 & & \langle \Sigma', D'\rangle
\end{tikzcd}$$

Multiset Databases

Object: $\langle \Sigma, E, e\rangle$
where Σ is a schema
and E is a set
and $e : E \to [\![\Sigma]\!]$
(i.e. E is the set of entries and e specifies their values)

Morphisms: $\langle f_\Sigma, f_E, f_e\rangle : \langle \Sigma, E, e\rangle \to \langle \Sigma', E', e'\rangle$
where
$f_\Sigma : \Sigma \to \Sigma'$
$f_E : E \to E'$

$$\begin{tikzcd}
E \arrow[r, "f_E"] \arrow[d, "e"'] \arrow[dr, phantom, "\underline{\underline{f_e}}"] & E' \arrow[d, "e'"] \\
{[\![\Sigma]\!]} \arrow[r, "{[\![f_\Sigma]\!]}"] & {[\![\Sigma']\!]}
\end{tikzcd}$$

Products of Multiset Databases

int	string

$\boxed{1} \times \boxed{\text{"hello"}} =$

1	"hello"

$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \times \boxed{\text{"hello"}} =$

1	"hello"
1	"hello"

$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \times \begin{bmatrix} \text{"hello"} \\ \text{"hello"} \end{bmatrix} =$

1	"hello"
1	"hello"
1	"
1	"

---

for (studentAddr ∈ Students × Address) if (student.Name = address.Name)

$$\begin{tikzcd} \langle \Sigma, D \rangle \arrow[rr, bend left, leftrightarrow, "\text{the same}"] \arrow[dr] & & \langle \Sigma, D' \rangle \arrow[dl] \\ & \langle \Sigma, ? \rangle & \end{tikzcd}$$

Fibres of Multiset Databases

$f : \langle \Sigma, E, e \rangle \to \langle \Sigma', E', e' \rangle$

consider only morphisms: $\langle \Sigma, E, e \rangle \to \langle \Sigma, E', e' \rangle$

such that Uf or $|f|$ $= id_\Sigma$

this category is called the fibre over Σ

Vertical Morphism in Multiset Databases over Σ

$\langle f_E, f_e \rangle : \langle E, e \rangle \longrightarrow \langle E', e' \rangle$

where $f_E : E \longrightarrow E'$

$$\begin{tikzcd} E \arrow[rr, "f_E"] \arrow[dr, "e"'] \arrow[rr, phantom, shift right=3ex, "\overset{f_e}{=}"] & & E' \arrow[dl, "e'"] \\ & \llbracket \Sigma \rrbracket & \end{tikzcd}$$

Fibred Products for Multiset Databases (over Σ)

$$\begin{tikzcd} & \bar{E} \arrow[d, dashed] \arrow[ddl, bend right] \arrow[ddr, bend left] & \\ & E \times_\Sigma E' \arrow[dl, "\pi_1"'] \arrow[dr, "\pi_2"] & \\ E \arrow[dr, "e"'] & & E' \arrow[dl, "e'"] \\ & \llbracket \Sigma \rrbracket & \end{tikzcd}$$

is called a pullback

$E \times_\Sigma E' = \{ (\epsilon, \epsilon') \mid \epsilon \in E \wedge \epsilon' \in E' \wedge e(\epsilon) = e(\epsilon') \}$

$\begin{bmatrix} \cancel{1,2} \\ \cancel{1,2} \\ \cancel{1,3} \\ \cancel{1,2} \\ \cancel{1,2} \\ \cancel{1,3} \\ 2,2 \\ 2,2 \\ \cancel{1,3} \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$\|$

$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \times_{int} \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$

Database Joins

$$\begin{tikzcd} \langle D \bowtie D', \Sigma \bowtie \Sigma' \rangle \arrow[r] \arrow[d] & \langle D', \Sigma' \rangle \\ \langle D, \Sigma \rangle & \end{tikzcd}$$

$$\begin{tikzcd} \Sigma \bowtie \Sigma' \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner"] & \Sigma' \arrow[d] \\ \Sigma \arrow[r] & \Sigma \cap \Sigma' \end{tikzcd}$$

σΓ a}

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for (d ∈ D)
    for (d' ∈ D')
        if (d.common = d'.common)
            <d.uncommon, d.common, d'.uncommon>
```