Hella Opfibrations Crunk

Comprehensions for Primitive Type Systems

Database Instance

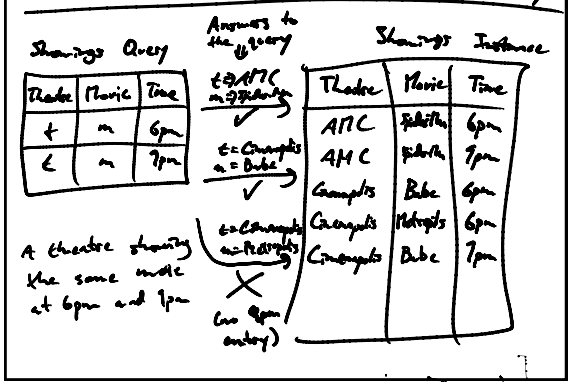
for(e in Employees) e.first+" "+e.last

SQL has types for int, string, and such but no "database" type

first-class

so how to formalize?

First: Database Queries as Databases - Toy/Real Theory



Category of Databases

Objects $\langle \Sigma, D \rangle$	Morphism $\langle f, \rho \rangle : \langle \Sigma, D \rangle \to \langle \Sigma', D' \rangle$
A schema Σ and a finite subset $D \subseteq \lVert \Sigma \rVert$ (i.e. the tuples in the database)	A schema morphism $f : \Sigma \to \Sigma'$ and a proof ρ that $\forall d \in D.\ \lVert f \rVert(d) \in D'$ (i.e. applying f to any tuple in D results in a tuple in D') where $\lVert f \rVert : \lVert \Sigma \rVert \to \lVert \Sigma' \rVert$

Underlying/Forgetful Functor

$|-|$ or U : Dat → Sch

$|\langle \Sigma, D \rangle| \longmapsto \Sigma$

$|\langle f, \rho \rangle| \longmapsto f$

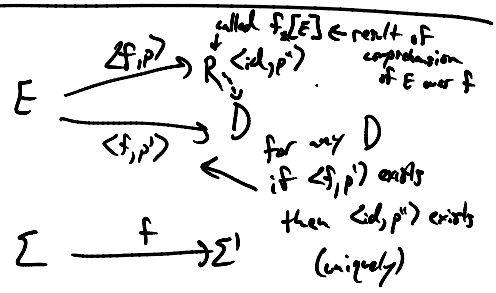
Back to Comprehensions

for (e in Empl) e.first + " " + e.last

(Empl: database; e.first + " " + e.last: morphism of schemas)

Dat	Empl	result of query
↑		
Sch:	$\lvert Empl \rvert$	String (e.first + " " + e.last)

(Cocartesian) Opcartesian Lifting



Opfibration

$$\begin{array}{c} D \\ \downarrow U \\ C \end{array}$$

U is an opfibration if

for all d object of D
and $f: Ud \to c$ for any c of C
there is an opcartesian lifting
of f
(called $f^*[d]$)