



Propagating Context

$$\frac{\rho_{ts} \dashv \rho' | c}{\bar{\rho}, \rho_{ts} \dashv \rho', \bar{\rho} | c}$$

Premonoidal Category: Extending Context

For every object C , there is an adjunction $C \otimes -$

$$f: A \rightarrow B \Rightarrow C \otimes f: C \otimes A \rightarrow C \otimes B$$

Symmetric: $\delta / \text{sym}: A \otimes B \rightarrow B \otimes A$
and is its own inverse

Propagating Context with ^{index} Effects

$$\rho_{ts} \dashv \rho' | c \Rightarrow f: A \rightarrow P_c B$$

$$\downarrow C \otimes$$

$$\bar{\rho}, \rho_{ts} \dashv \bar{\rho}', \rho' | c \quad C \otimes f: C \otimes A \rightarrow C \otimes P_c B \xrightarrow{\text{strength}} P_c(C \otimes B)$$

(Lax) Premonoidal Functor

Monoidal Categories

$$f: A \rightarrow A' \quad g: B \rightarrow B'$$

$$A \otimes B \xrightarrow{f \otimes g} A' \otimes B \xrightarrow{A' \otimes g} A' \otimes B'$$

$$\parallel$$

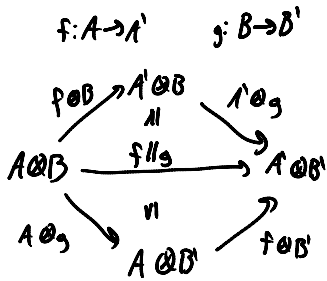
$$A \otimes B \xrightarrow{A \otimes g} A \otimes B' \xrightarrow{f \otimes B'} A' \otimes B'$$

Ordering on Computations

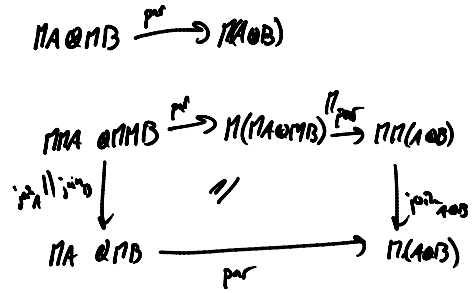
$$f, g: A \rightarrow B$$

$f \leq g$ if f can be used whenever g can be used

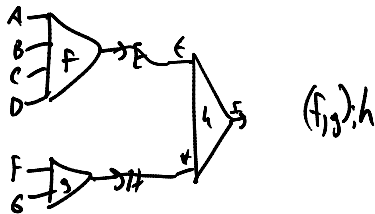
Productable Category



Productable Objects



Multicategories



Multicategory \rightarrow Monoidal Category

Objects: Lists of objects of the multicategory

\otimes : concatenation of lists

Morphisms: Lists of morphisms of the multicategory

the list $\begin{matrix} \triangleleft \\ \text{F} \\ \text{D} \end{matrix}$ will be from $[A, B, C]$ to $[D, E]$

Monoidal \rightarrow Multicategory

Objects: objects of the monoidal category

Morphism $A, B, C \rightarrow E$: a morphism from $A \otimes B \otimes C \rightarrow E$

