Effectors and Productors

General-Purpose Effect Rules

$$\frac{\Gamma \vdash s \dashv \Gamma' \mid \varepsilon \qquad \Gamma' \vdash s' \dashv \Gamma'' \mid \varepsilon' \qquad \varepsilon ; \varepsilon' = \varepsilon''}{\Gamma \vdash s ; s' \dashv \Gamma'' \mid \varepsilon''} \text{ Sequencing}$$

$$\frac{\mathbb{1} = \varepsilon}{\Gamma \vdash \dashv \Gamma \mid \varepsilon} \text{ Empty Pure} \qquad \frac{\Gamma \vdash s \dashv \Gamma' \mid \varepsilon \qquad \varepsilon \le \varepsilon'}{\Gamma \vdash s \dashv \Gamma' \mid \varepsilon'} \text{ Subeffects}$$

Principalled Effectoid

Partial Monoid $(\mathrm{Eff}, \mathbb{1}, ;)$ Identity, Associative

$$\exists \bar\varepsilon.\ \varepsilon_1 ; \varepsilon_2 = \bar\varepsilon \wedge \bar\varepsilon ; \varepsilon_3 = \varepsilon \Leftrightarrow \exists \hat\varepsilon.\ \varepsilon_2 ; \varepsilon_3 = \hat\varepsilon \wedge \varepsilon_1 ; \hat\varepsilon = \varepsilon$$

Congruent Preorder $\le$

$\varepsilon \le \varepsilon$ $\qquad$ $\varepsilon \le \varepsilon' \wedge \varepsilon' \le \varepsilon'' \Rightarrow \varepsilon \le \varepsilon''$

$\varepsilon_1 \le \varepsilon_1'$, $\varepsilon_2 \le \varepsilon_2'$, $\varepsilon \le \varepsilon'$ $\Rightarrow$ $\varepsilon_1 ; \varepsilon_2 = \varepsilon \Rightarrow \varepsilon_1' ; \varepsilon_2' = \varepsilon'$

Producer Effect Semantics: Productoid

For each effect $\varepsilon \in \mathrm{Eff}$, a functor P_ε

For each $\mathbb{1} = \varepsilon$, a nat trans $\mathrm{unit}_\varepsilon : \mathrm{Id} \Rightarrow P_\varepsilon$

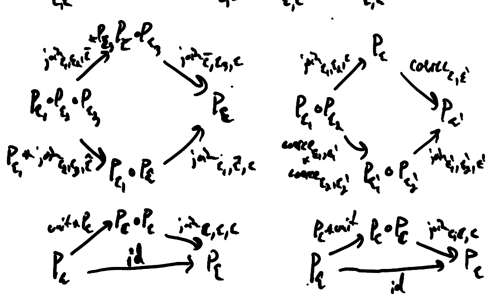
For each $\varepsilon_1 ; \varepsilon_2 = \varepsilon$, a nat trans $\mathrm{join}_{\varepsilon_1,\varepsilon_2,\varepsilon} : P_{\varepsilon_1} \circ P_{\varepsilon_2} \Rightarrow P_\varepsilon$

For each $\varepsilon \le \varepsilon'$, a nat trans $\mathrm{coerce}_{\varepsilon,\varepsilon'} : P_\varepsilon \Rightarrow P_{\varepsilon'}$

satisfying equations

Equational Requirements

$\mathrm{coerce}_{\varepsilon,\varepsilon} = \mathrm{id}$ $\qquad$ $\mathrm{coerce}_{\varepsilon,\varepsilon'} ; \mathrm{coerce}_{\varepsilon',\varepsilon''} = \mathrm{coerce}_{\varepsilon,\varepsilon''}$



Non-Compositional Effect System

$x := *p$ | read

$*p := x$ | write

compositional

$x := *p \, ; \, *p := x$ | pure $\le$ read/write

non-compositional

Analytized Effect Systems

A join semilattice of effects $\langle Eff, \bot, \sqcup \rangle$

how do I get a principled effectoid?

$$\varepsilon = \bot \qquad \mathring{;} = \sqcup$$

Properties?

Total ($\mathring{;}$) Idempotent ($c \mathring{;} c = c$)

Centric ($\varepsilon \leq c$) Increasing ($c \mathring{;} c' = c'' \Rightarrow c, c' \leq c''$)