



Linear Types for Locking

L: Type (a lock)      the environment controls  
 C: Type (a capability)      how L is passed around

acquire:  $L + C$       disallowing cut on L

release:  $C + L$        $L, \delta + L, \delta$        $L, \delta + L, \delta$

get:  $C + C \otimes Z$        $Z + \delta, L \multimap$        $\delta + \delta, L \multimap$

set:  $C \otimes Z + C$        $Z + \delta, ?(L \multimap)$        $\delta + \delta, ?(L \multimap)$

$Z + \delta, ?(L \multimap), ?(L \multimap)$   
 $Z + \delta, ?(L \multimap)$

Ordered Types for RegExes

$a^* b^* \leq (ab)^*$

$\frac{A? \otimes B? + (A \otimes B)?}{A?, B? + (A \otimes B)?}$

$\frac{A?, D? + (A \otimes D)?, (A \otimes D)?}{A? + (A \otimes D)? \quad B? + (A \otimes D)?}$  (\*)

$\frac{A? + (A \otimes D)? \quad B? + (A \otimes D)?}{A + (A \otimes D)? \quad B + (A \otimes D)?}$

$\frac{A + A \otimes \quad B + A \otimes}{A + A \quad B + B}$

Classical Logic

$P \vdash \Delta \Rightarrow !P \vdash \Delta$

cut out  
 $\downarrow$

$\frac{P \vdash \Delta, A \quad A, P' \vdash \Delta'}{P, P' \vdash \Delta, \Delta'} \Rightarrow !P \vdash \Delta, A \quad A, !P' \vdash \Delta'$

Classical Logic - Attempt 2 = Strict

$P \vdash \Delta \Rightarrow !P \vdash ?\Delta$        $\Downarrow$

$\frac{P \vdash \Delta, A \quad A, P' \vdash \Delta'}{P, P' \vdash \Delta, \Delta'} \Rightarrow \frac{!P \vdash ?\Delta, ?A \quad A, !P' \vdash ?\Delta'}{?A, !P' \vdash ?\Delta'}$

$!P, !P' \vdash ?\Delta, ?\Delta'$

Classical Logic - Attempt 2 = Lazy

$P \vdash \Delta \Rightarrow !?P \vdash \Delta$

$\frac{P \vdash \Delta, A \quad A, P' \vdash \Delta'}{P, P' \vdash \Delta, \Delta'} \Rightarrow \frac{!?P \vdash ?\Delta, ?A \quad !?A, !?P' \vdash ?\Delta'}{!?P \vdash ?\Delta, !?A}$

$!?P, !?P' \vdash ?\Delta, ?\Delta'$