Assignment 5

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Definition. The category $\mathbf{Set} \times \mathbf{Set}$ has pairs of sets $\langle A, B\rangle$ as its objects, and has pairs of functions $\langle f : A \to A', g : B \to B'\rangle$ as its morphisms from $\langle A, B\rangle$ to $\langle A', B'\rangle$. The identity morphism on $\langle A, B\rangle$ is the pair of functions $\langle id_A, id_B\rangle$. The composition $\langle f, g\rangle\,;\langle f', g'\rangle$ is the pair of functions $\langle f\,; f', g\,; g'\rangle$.

Exercise 1. Show that the function on objects mapping a pair of sets $\langle A, B\rangle$ to the set of pairs $A \times B$ extends to a functor from $\mathbf{Set} \times \mathbf{Set}$ to $\mathbf{Set}$. This functor is often denoted $\mathbf{Set} \times \mathbf{Set} \xrightarrow{\times} \mathbf{Set}$.

Exercise 2. Show that $\mathbf{Set} \times \mathbf{Set} \xrightarrow{\times} \mathbf{Set}$ is neither full nor faithful.

Exercise 3. Show that there is a concrete functor (over $\mathbf{Set}$) from $\mathbf{Set}$ to $\mathbf{Rel}(2)$ that is finer than all other such concrete functors.