Preface

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1 Metatheory

Although we will be using *The Joy of Cats* as our primary reference, we will stray from it in a few ways. One such deviation is our metatheory. A metatheory is the global notion of truth that we are operating in. The high-level and self-descriptive nature of category theory makes metatheoretical issues arise surprisingly often. In *The Joy of Cats*, this is evidenced by the fact that Section 2 is devoted solely to metatheory, setting up sets, classes (of sets), and conglomerates (of classes).

To make matters simpler, we will conceptually be assuming a set of all sets. This is known to be inconsistent, meaning it makes everything false and true at the same time, so technically we will be using an infinite heirarchy of set universes. However, we will be using this hierarchy implicitly, so only in rare cases will there be any explicit mention of universe levels.

Lastly, unless noted otherwise, we will be using a constructive metatheory. This means we will not prove something holds by proving that it is impossible for it not to hold. Although this occasionally means doing proofs a little differently, it generally forces proofs to be more informative and makes connections more apparent.

2 Notation

We will for the most part use the same notation as *The Joy of Cats*. One important deviation, though, is **Rel**. In *The Joy of Cats*, **Rel** denotes the category where objects are binary relations and morphisms are relation-preserving functions. However, elsewhere **Rel** usually denotes the very useful category where objects are sets and a morphism from X to Y is a relation on X and Y, i.e. a subset of $X \times Y$. We will use **Rel** to denote the latter, and we will use **Rel**(2) to denote the former.

A second deviation regards composition. Given a morphism $f : A \to B$ and a morphism $g : B \to C$, *The Joy of Cats* denotes the composition of f and g with $g \circ f : A \to C$. While we will also use this notation, we will more often prefer the alternative notation $f \mathbin{;} g : A \to C$ since this has the benefit that the order of f and g is maintained when reading them diagramatically. That is, the composition of $A \xrightarrow{f} B \xrightarrow{g} C$ can then be denoted is $A \xrightarrow{f \,;\, g} C$, with f preceding g in both cases.