Assignment 9

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Exercise 1. Classify the initial morphisms of **Set** viewed as a concrete category over **Rel**. That is, prove that a function is initial if and only if it belongs to a common class of functions.

Exercise 2. Let $E : \mathbf{Set} \to \mathbf{Rel}(2)$ be the functor mapping each set A to the binary relation $\langle A, =_A \rangle$, where $=_A$ is the equality relation on A, and maps each function $f : A \to B$ to itself, taking advantage of the fact that all functions map equal elements to equal elements. Describe the mapping on objects of the left adjoint of E, the corresponding transposition operators $\rightarrow$ and $\leftarrow$, and the corresponding unit η and counit ε of the adjunction. Do not describe the mapping on morphisms of the left adjoint of E, and do not prove the many equational requirements of the corresponding transposition or adjunction.

As an aid, you can take advantage of the fact that the full subcategory **Equiv** of $\mathbf{Rel}(2)$ comprised of equivalence relations is concretely reflective over **Set**, with the reflector $\mathrm{Eq} : \mathbf{Rel}(2) \to \mathbf{Equiv}$ mapping each relation to its equivalence closure, i.e. its reflexive, symmetric, and transitive closure.

Exercise 3. Show that pullback squares compose. That is, in the diagram below, show that if $\langle A_1, B_1, A_2, B_2 \rangle$ and $\langle A_2, B_2, A_3, B_3 \rangle$ are pullback squares, then so is $\langle A_1, B_1, A_3, B_3 \rangle$.