Assignment 7

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Exercise 1. Given functors \( A_1 \xrightarrow{F_1} B \xleftarrow{F_2} A_2 \), recall that there are functors \( A_1 \xleftarrow{\pi_1} F_1 \downarrow F_2 \xrightarrow{\pi_2} A_2 \). Show that there is a natural transformation from \( F_1 \downarrow F_2 \xrightarrow{\pi_1} \) to \( F_1 \downarrow F_2 \xrightarrow{\pi_2} B \).

Exercise 2. singleton : Set \( \Rightarrow \) \( L \) and flatten : \( LL \Rightarrow L \) are two particularly important natural transformations pertaining to lists. The following equate various compositions of these natural transformations:

1. \((\text{singleton} \ast L) \circ \text{flatten} = \text{id}_L^L\)
2. \((L \ast \text{singleton}) \circ \text{flatten} = \text{id}_L^L\)
3. \((\text{flatten} \ast L) \circ \text{flatten} = (L \ast \text{flatten}) \circ \text{flatten}\)

Write each side of equations 1, 2, and 3 as a polymorphic program of the form \( \lambda l : \tau. \ldots \) for some type \( \tau \) (referencing a type variable \( \alpha \)) and using the following “library” functions with explicit subscripts (i.e. use explicit type arguments):

\[
\text{map}_{\alpha, \beta} : (\alpha \rightarrow \beta) \rightarrow (L\alpha \rightarrow L\beta)
\]
\[
\text{singleton}_\alpha : \alpha \rightarrow L\alpha
\]
\[
\text{flatten}_\alpha : LL\alpha \rightarrow L\alpha
\]

Then prove equalities 1 and 2 (but not 3) by stepping through how they act on lists of the form \([x_1, \ldots, x_n]\).

Exercise 3. Prove that \( \text{Prost} \) is in fact a 2-thin 2-category, with \( f \leq g : \langle A, \leq_A \rangle \rightarrow \langle B, \leq_B \rangle \) defined as \( \forall a \in A. f(a) \leq_B g(a) \). You may assume that \( \text{Prost} \) is a category, and because it is 2-thin you do not need to prove the required equalities between various 2-cells because they all hold automatically.