Assignment 7

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Exercise 1. Given functors $\mathbf{A}_1 \xrightarrow{F_1} \mathbf{B} \xleftarrow{F_2} \mathbf{A}_2$, recall that there are functors $\mathbf{A}_1 \xleftarrow{\pi_1} F_1 \downarrow F_2 \xrightarrow{\pi_2} \mathbf{A}_2$. Show that there is a natural transformation from $F_1 \downarrow F_2 \xrightarrow{\pi_1\,;\,F_1} \mathbf{B}$ to $F_1 \downarrow F_2 \xrightarrow{\pi_2\,;\,F_2} \mathbf{B}$.

Exercise 2. $\texttt{singleton} : \mathbf{Set} \Rightarrow \mathbb{L}$ and $\texttt{flatten} : \mathbb{L}\mathbb{L} \Rightarrow \mathbb{L}$ are two particularly important natural transformations pertaining to lists. The following equate various compositions of these natural transformations:

1. $(\texttt{singleton} * \mathbb{L})\,;\texttt{flatten} = id_{\mathbb{L}}$
2. $(\mathbb{L} * \texttt{singleton})\,;\texttt{flatten} = id_{\mathbb{L}}$
3. $(\texttt{flatten} * \mathbb{L})\,;\texttt{flatten} = (\mathbb{L} * \texttt{flatten})\,;\texttt{flatten}$

Write each side of equations 1, 2, *and* 3 as a polymorphic program of the form $\lambda \texttt{l} : \tau.\ \ldots$ for some type τ (referencing a type variable α) and using the following "library" functions *with explicit subscripts* (i.e. use explicit type arguments):

$\texttt{map}_{\alpha,\beta} : (\alpha \to \beta) \to (\mathbb{L}\alpha \to \mathbb{L}\beta)$

$\texttt{singleton}_{\alpha} : \alpha \to \mathbb{L}\alpha$

$\texttt{flatten}_{\alpha} : \mathbb{L}\mathbb{L}\alpha \to \mathbb{L}\alpha$

Then prove equalities 1 and 2 (but not 3) by stepping through how they act on lists of the form $[x_1, \ldots, x_n]$.

Exercise 3. Prove that **Prost** is in fact a 2-thin 2-category, with $f \leq g : \langle A, \leq_A \rangle \to \langle B, \leq_B \rangle$ defined as $\forall a \in A.\ f(a) \leq_B g(a)$. You may assume that **Prost** is a category, and because it is 2-thin you do not need to prove the required equalities between various 2-cells because they all hold automatically.