Assignment 5

Ross Tate

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Exercise 1. Given a monoid \(\langle A, e, *\rangle\), an element \(z \in A\) is said to be an absorbing element (also known as a zero element) if the following holds:

\[
\forall a \in A. \quad z * a = z = a * z
\]

An example is the natural number 0 for the monoid \(\langle \mathbb{N}, 1, *\rangle\). A monoid homomorphism is said to preserve absorbing elements if it maps absorbing elements to absorbing elements.

Absorbing elements of a monoid are provably unique (if they exist). Consequently, the category, say \(\text{Mon}_0\), of monoids with absorbing elements and absorbing-element-preserving monoid homomorphisms is a subcategory of \(\text{Mon}\). Prove that it is a reflective subcategory, but skip the tedious proof that the object in \(\text{Mon}_0\) that you define in fact satisfies the identity, associativity, and absorbing equalities, as well as the tedious proof that the reflection arrow you define is in fact a monoid homomorphism.

Exercise 2. Given subcategories \(A_1\) and \(A_2\) of a category \(B\), recall that \(A_1 \cap A_2\) is the subcategory of \(B\) comprised of the objects and morphisms contained in both \(A_1\) and \(A_2\). Suppose \(A_1\) is a reflective subcategory of \(B\), and suppose \(A_1 \cap A_2\) is a full subcategory of \(A_1\). What simple additional property of the reflection arrows is sufficient (though not necessary) for \(A_1 \cap A_2\) to be a reflective subcategory of \(A_2\)? Prove that it is sufficient.