Assignment 1

Ross Tate

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Exercise 1. Every category describes a (directed) graph; the vertices of the graph are the objects of the category, and the edges of the graph are the morphisms of the category. Explain at a high level why two of the graphs shown in Exercise 3A(c) cannot be described by any category. Note that in all the graphs the self-loops that would correspond to identity morphisms have been omitted for readability.

Exercise 2. Given two sets A and B and two binary relations $R \subseteq A \times A$ and $S \subseteq B \times B$, a function $f : A \to B$ is said to be relation-*reflecting* (as opposed to relation-*preserving*) if f mapping two inputs to related outputs implies the two inputs are themselves related, i.e. if $\forall a, a' \in A.\ f(a)\ S\ f(a') \implies a\ R\ a'$ holds. For example, $\lambda x.\ 0$ *is* a relation-*preserving* function from $\langle \mathbb{Z}, \leq \rangle$ to itself, but it is *not* relation-*reflecting*. On the other hand, $\lambda x.\ 0$ is *not* a relation-*preserving* function from $\langle \mathbb{Z}, < \rangle$ to itself, but it *is* relation-*reflecting*. Show that there is a well-defined category of relation-reflecting functions (in which $\lambda x.\ 0$ is an endomorphism on $\langle \mathbb{Z}, < \rangle$ but not on $\langle \mathbb{Z}, \leq \rangle$).

Exercise 3. Suppose you have a set of objects, a set of morphisms between each two objects, and an associative composition operator on those morphisms. Show that there is at most one category with those components. Note that Exercise 3D(a) in *The Joy of Cats* provides an alternative formulation of this same exercise.