Assignment 6

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March 7, 2018

Exercise 1. **Grp** is a full subcategory of **Mon**. Prove that it is also a coreflective subcategory of **Mon**, but skipping the tedious proofs that the coreflection object satisfies identity, associative, and inverse equalites, that the coreflection arrow is a monoid homomorphism, and that the induced morphism is unique and a group homomorphism.

Exercise 2. A concretely reflective subcategory of a concrete category $\mathbf{C} \xrightarrow{U} \mathbf{X}$ is a subcategory $\mathbf{S}$ of $\mathbf{C}$ with reflection arrows $\{r_C\}_{C \in \mathbf{C}}$ satisfying the additional property that every reflection arrow is *identity-carried*, meaning the following holds:

$$\forall C \in \mathbf{C}.\, U(r_C) = id_{UC}$$

For example, **Prost** is a concretely reflective subconstruct of $\mathbf{Rel}(2)$, but **Pos** is not (even though it is a reflective subcategory of $\mathbf{Rel}(2)$).

The dual of a concrete category $\mathbf{C} \xrightarrow{U} \mathbf{X}$ is the concrete category $\mathbf{C}^{\mathrm{op}} \xrightarrow{U^{\mathrm{op}}} \mathbf{X}^{\mathrm{op}}$. A concretely *co*reflective subcategory is the dual of a concretely reflective subcategory. Give the direct definition of a concretely coreflective subcategory and give a non-trivial example (meaning an example where the subcategory is *proper*, i.e. either not wide or not full).

Exercise 3. Recall that $\mathbb{P} : \mathbf{Set} \to \mathbf{Set}$ is the functor mapping each set to its powerset and each function to the function mapping subsets to their image. Given a coalgebra of $\langle X, c : X \to \mathbb{P}X \rangle$, one can define the relation $x \; R_c \; x'$ as $x' \in c(x)$. This extends to a concrete isomorphism between $\mathbf{Coalg}(\mathbb{P})$ and a wide subconstruct (i.e. a wide subcategory) of $\mathbf{Rel}(2)$. Describe that wide subcategory without referring to powersets or subsets. That is, describe what additional property a morphism of $\mathbf{Rel}(2)$ must satisfy so that it corresponds to a morphism of $\mathbf{Coalg}(\mathbb{P})$, but describe it using propositional formulae rather than powersets and subsets.