Assignment 5

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March 1, 2018

Exercise 1. Given a monoid $\langle A, e, * \rangle$, an element $z \in A$ is said to be an *absorbing element* (also known as a *zero element*) if the following holds:

$$\forall a \in A.\ z * a = z = a * z$$

An example is the natural number 0 for the monoid $\langle \mathbb{N}, 1, * \rangle$. A monoid homomorphism is said to preserve absorbing elements if it maps absorbing elements to absorbing elements.

Absorbing elements of a monoid are provably unique (if they exist). Consequently, the category, say $\mathbf{Mon}_0$, of monoids with absorbing elements and absorbing-element-preserving monoid homomorphisms is a subcategory of $\mathbf{Mon}$. Prove that it is a reflective subcategory, but skip the tedious proof that the object in $\mathbf{Mon}_0$ that you define in fact satisfies the identity, associativity, and absorbing equalities, as well as the tedious proof that the reflection arrow you define is in fact a monoid homomorphism.

Exercise 2. Given subcategories $\mathbf{A}_1$ and $\mathbf{A}_2$ of a category $\mathbf{B}$, recall that $\mathbf{A}_1 \cap \mathbf{A}_2$ is the subcategory of $\mathbf{B}$ comprised of the objects and morphisms contained in both $\mathbf{A}_1$ and $\mathbf{A}_2$. Suppose $\mathbf{A}_1$ is a reflective subcategory of $\mathbf{B}$, and suppose $\mathbf{A}_1 \cap \mathbf{A}_2$ is a full subcategory of $\mathbf{A}_1$. What simple additional property of the reflection arrows is sufficient (though not necessary) for $\mathbf{A}_1 \cap \mathbf{A}_2$ to be a reflective subcategory of $\mathbf{A}_2$? Prove that it is sufficient.