Exercise 1. Prove that the 2-category MULTICAT is powered. I am content if you can construct the power multicategory $E \triangleleft M$ and the necessary functor $E \to M_{\text{MULTICAT}}(E \triangleleft M, M)$ and show that any multicategory $D$ with a functor $E \to M_{\text{MULTICAT}}(D, M)$ has a 1-cell from $D$ to $E \triangleleft M$. The remaining requirements need to hold but do not need to be proved.

Proof. Let $C_M$ be the category whose objects are the same as $M$ and whose morphisms are the unary morphisms of $M$ with the obvious identity and composition. Suppose $F_1, \ldots, F_n$ and $G$ are functors from $E$ to $C_M$, then let a multitransformation $\alpha$ from $\vec{F}$ to $G$ map an object $E$ of $E$ to morphism of $M$ from $[F_1(E), \ldots, F_n(E)]$ to $G(E)$, and let $\alpha$ be natural if for every morphism $e : E \to E'$ of $E$ the composition $\alpha_{E'} : G(e)$ equals the composition $[F_1(e), \ldots, F_n(e)] : \alpha_{E'}$.

Define $E \triangleleft M$ to be the category whose objects are functors from $E$ to $C_M$ and whose morphisms are natural multitransformations with composition and identity each defined pointwise (which obviously always results in a natural multitransformation). Given an object $E$ of $E$, let $\pi_E : E \triangleleft M \to M$ map the functor $F : E \to C_M$ to the object $F(E)$ and the multitransformation $\alpha$ to the morphism $\alpha_E$, which defines a functor of multicategories because composition and identity in $E \triangleleft M$ are defined pointwise. Given a morphism $e : E \to E'$ of $E$, let $\pi_e : \pi_E \Rightarrow \pi_{E'}$ be the transformation mapping a functor $F : E \to C_M$ to the morphism $F(e) : [F(E)] \to F(E')$, which is natural due to the naturality requirement on the morphisms of $E \triangleleft M$. $\pi : E \to M_{\text{MULTICAT}}(E \triangleleft M, M)$ is functorial because each object of $E \triangleleft M$ is functorial.