Databases

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Let **Fin** be the full subcategory of **Set** whose objects are the finite sets. Let $F : \mathbf{Fin} \to \mathbf{Set}$ be the inclusion functor. Define **Dat** to be $F \downarrow \mathbf{Set}$. Let $U : \mathbf{Dat} \to \mathbf{Set}$ be the right projection for the comma category.

The intuition is that **Dat** represents the category of databases. An object $I \xrightarrow{d} X$ represents a database of X values; the I represents the finite set of entries, and d specifies the X-value of each entry. A morphism $\langle i : I \to J, f : X \to Y, \centerdot \rangle$ represents applying the computation f to each entry to get a corresponding entry in the target database, where the corresponding entry is specified by i. In particular, if f is an identity function, then the function i shows that the entries of the source database are a subset of the entries of the target database.

Exercise 1. Prove that U is an opfibration.

Proof. Given an object $I \xrightarrow{d} X$ and a function $f : X \to Y$, let the lifting of Y be $I \xrightarrow{d\,;f} Y$ and the lifting of f be $\langle id, f, \centerdot \rangle$. To prove $\langle id, f, \centerdot \rangle$ is opcartesian, suppose there is a morphism $\langle i, f', \centerdot \rangle : (I \xrightarrow{d} X) \to (I' \xrightarrow{d'} X')$ and a function $g : Y \to X'$ such that $f\,;g$ equals f'. Then $\langle i, g, \centerdot \rangle$ is a lifting of g with the property that $\langle id, f, \centerdot \rangle\,;\langle i, g, \centerdot \rangle$ equals $\langle i, f', \centerdot \rangle$. For uniqueness, suppose $\langle i', g', \centerdot \rangle$ is also a lifting of g with the property that $\langle id, f, \centerdot \rangle\,;\langle i', g', \centerdot \rangle$ equals $\langle i, f', \centerdot \rangle$. To be a lifting of g, g' must equal g, and for the equality to hold, $id\,;i'$ must equal i, which implies i' equals i. Thus, $\langle i', g', \centerdot \rangle$ equals $\langle i, g, \centerdot \rangle$. $\square$