Let \( \textbf{Fin} \) be the full subcategory of \( \textbf{Set} \) whose objects are the finite sets. Let \( F : \textbf{Fin} \to \textbf{Set} \) be the inclusion functor. Define \( \textbf{Dat} \) to be \( F \downarrow \textbf{Set} \). Let \( U : \textbf{Dat} \to \textbf{Set} \) be the right projection for the comma category.

The intuition is that \( \textbf{Dat} \) represents the category of databases. An object \( I \xrightarrow{d} X \) represents a database of \( X \) values; the \( I \) represents the finite set of entries, and \( d \) specifies the \( X \)-value of each entry. A morphism \( \langle i : I \to J, f : X \to Y, \cdot \rangle \) represents applying the computation \( f \) to each entry to get a corresponding entry in the target database, where the corresponding entry is specified by \( i \). In particular, if \( f \) is an identity function, then the function \( i \) shows that the entries of the source database are a subset of the entries of the target database.

**Exercise 1.** Prove that \( U \) is an opfibration.

*Proof.* Given an object \( I \xrightarrow{d} X \) and a function \( f : X \to Y \), let the lifting of \( Y \) be \( I \xrightarrow{d,f} Y \) and the lifting of \( f \) be \( \langle \text{id}, f, \cdot \rangle \). To prove \( \langle \text{id}, f, \cdot \rangle \) is opcartesian, suppose there is a morphism \( \langle i, f', \cdot \rangle : (I \xrightarrow{d} X) \to (I' \xrightarrow{d'} X') \) and a function \( g : Y \to X' \) such that \( f \circ g \) equals \( f' \). Then \( \langle i, g, \cdot \rangle \) is a lifting of \( g \) with the property that \( \langle \text{id}, f, \cdot \rangle ; \langle i, g, \cdot \rangle \) equals \( \langle i, f', \cdot \rangle \). For uniqueness, suppose \( \langle i', g', \cdot \rangle \) is also a lifting of \( g \) with the property that \( \langle \text{id}, f, \cdot \rangle ; \langle i', g', \cdot \rangle \) equals \( \langle i, f', \cdot \rangle \). To be a lifting of \( g \), \( g' \) must equal \( g \), and for the equality to hold, \( \text{id} : i' \) must equal \( i \), which implies \( i' \) equals \( i \). Thus, \( \langle i', g', \cdot \rangle \) equals \( \langle i, g, \cdot \rangle \). \( \square \)