Productors

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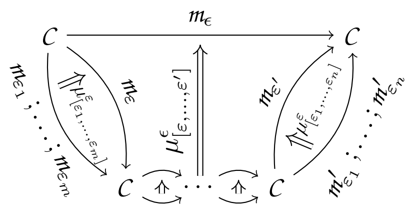
Definition (Productor of an Effector $\langle E, \overset{\fatsemi}{\mapsto}, \centerdot, \centerdot \rangle$ for a 2-Category $\mathbf{C}$)**.** A tuple $\langle \mathcal{C}, m, \mu, \mathfrak{a}, \mathfrak{i} \rangle$ whose components have the following types:

Object $\mathcal{C}$ is an object of $\mathbf{C}$

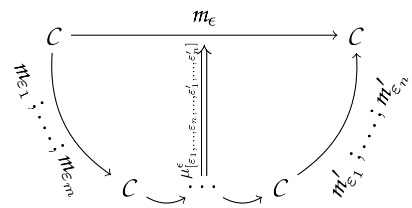
Morphisms m maps each $\varepsilon : E$ to a morphism m_ε of $\mathbf{C}$ from $\mathcal{C}$ to $\mathcal{C}$

Join μ maps each related pair $[\varepsilon_1, \ldots, \varepsilon_n] \overset{\fatsemi}{\mapsto} \varepsilon$ to a 2-cell $\mu^{\varepsilon}_{[\varepsilon_1,\ldots,\varepsilon_n]}$ of $\mathbf{C}$ from $m_{\varepsilon_1} ; \ldots ; m_{\varepsilon_n} \Rightarrow m_\varepsilon$

Associativity $\mathfrak{a}$ proves



equals



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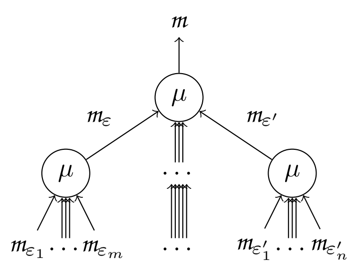
In other words,



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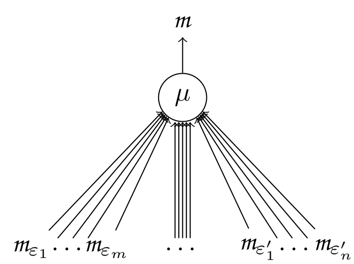

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Identity $\mathfrak{i}$ is a proof that $\forall \varepsilon : E.\ \mu^{\varepsilon}_{[\varepsilon]} = id_{m_\varepsilon} : m_\varepsilon \Rightarrow m_\varepsilon$

Exercise 1. Prove that a monad is a productor for the effector with one element and with $\overset{\fatsemi}{\mapsto}$ always true.

Exercise 2. Every effector corresponds to a thin multicategory, which in turn corresponds to an opetory with 1 object and at most one 2-cell from any domain to any codomain. Prove that a productor for an effector is simply a functor from the corresponding opetory.

Definition (Productoid of an Effectoid $\langle E, \mathfrak{E} \mapsto \bullet, \leq, \bullet\fatsemi\bullet \mapsto \bullet, \centerdot \rangle$ for a 2-Category $\mathbf{C}$)**.** A tuple $\langle \mathcal{C}, m, \mu_{\mathfrak{E}}, \mu_{\leq}, \mu_{\fatsemi}, \mathfrak{c} \rangle$ whose components have the following types:

Object $\mathcal{C}$ is an object of $\mathbf{C}$

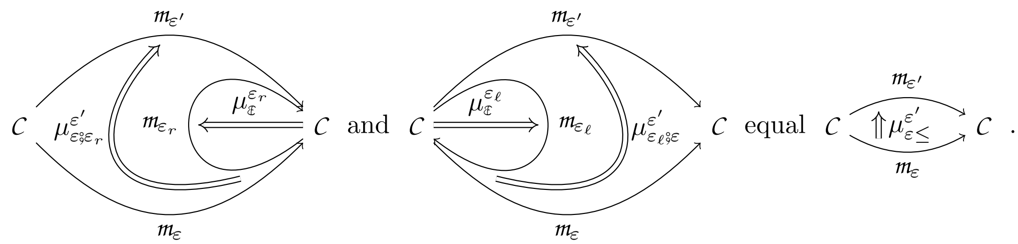
Morphisms m maps each $\varepsilon : E$ to a morphism m_ε of $\mathbf{C}$ from $\mathcal{C}$ to $\mathcal{C}$

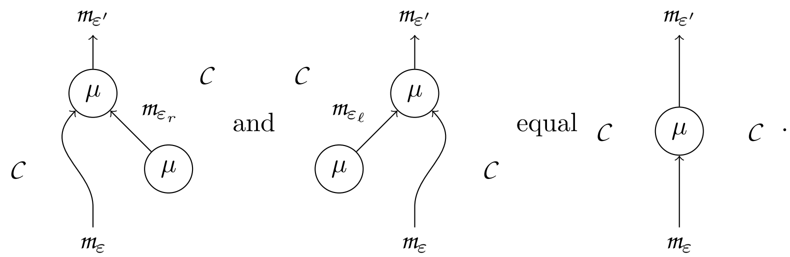
Unit $\mu_{\mathfrak{E}}$ maps each $\varepsilon : E$ satisfying $\mathfrak{E} \mapsto \varepsilon$ to a 2-cell $\mu^{\varepsilon}_{\mathfrak{E}}$ of $\mathbf{C}$ from $id_{\mathcal{C}} \Rightarrow m_\varepsilon$

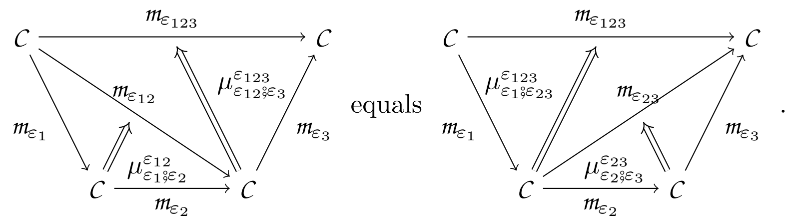
Coercion $\mu_{\leq}$ maps each $\varepsilon, \varepsilon' : E$ satisfying $\varepsilon \leq \varepsilon'$ to a 2-cell $\mu^{\varepsilon'}_{\varepsilon\leq}$ of $\mathbf{C}$ from $m_\varepsilon \Rightarrow m_{\varepsilon'}$

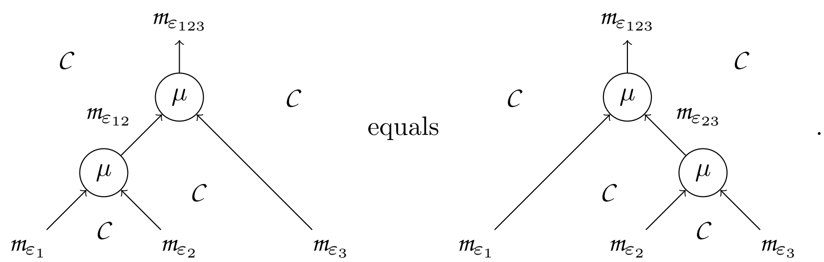
Join $\mu_{\fatsemi}$ maps each $\varepsilon_1, \varepsilon_2, \varepsilon : E$ satisfying $\varepsilon_1\fatsemi\varepsilon_2 \mapsto \varepsilon$ to a 2-cell $\mu^{\varepsilon}_{\varepsilon_1\fatsemi\varepsilon_2}$ of $\mathbf{C}$ from $m_{\varepsilon_1} ; m_{\varepsilon_2} \Rightarrow m_\varepsilon$

Coherence $\mathfrak{c}$ is a proof that $\mu^{\varepsilon}_{\varepsilon\leq}$ always equals $\mathit{id}_{m_\varepsilon}$ and the following equalities all hold whenever well defined:

First, 

In other words, 

Second, 

In other words, 

Third, $\mathit{id}_{\mathcal{C}} \overset{\mu^{\varepsilon}_{\mathfrak{e}}}{\Longrightarrow} m_\varepsilon \overset{\mu^{\varepsilon'}_{\varepsilon\leq}}{\Longrightarrow} m_{\varepsilon'}$ equals $\mathit{id}_{\mathcal{C}} \overset{\mu^{\varepsilon'}_{\mathfrak{e}}}{\Longrightarrow} m_{\varepsilon'}$.

Fourth, $m_{\varepsilon_1} ; m_{\varepsilon_2} \overset{\mu^{\varepsilon}_{\varepsilon_1 ⨾ \varepsilon_2}}{\Longrightarrow} m_\varepsilon \overset{\mu^{\varepsilon'}_{\varepsilon\leq}}{\Longrightarrow} m_{\varepsilon'}$ equals $m_{\varepsilon_1} ; m_{\varepsilon_2} \overset{\mu^{\varepsilon'}_{\varepsilon_1 ⨾ \varepsilon_2}}{\Longrightarrow} m_{\varepsilon'}$.

Theorem. *If an effector is semi-strict, then there is a bijection between the set of productors for that effector and the set of productoids for the effectoid corresponding to that semi-strict effector. The bijection preserves the object $\mathcal{C}$ and the mapping m. The 2-cells $\mu^{\varepsilon}_{\mathfrak{e}}$ correspond to the 2-cells $\mu^{\varepsilon}_{[\,]}$; the 2-cells $\mu^{\varepsilon'}_{\varepsilon\leq}$ correspond to the 2-cells $\mu^{\varepsilon'}_{[\varepsilon]}$; and the 2-cells $\mu^{\varepsilon}_{\varepsilon_1 ⨾ \varepsilon_2}$ correspond to the 2-cells $\mu^{\varepsilon}_{[\varepsilon_1,\varepsilon_2]}$.*

Definition (Postmodule of a Productor)**.** A tuple $\langle \mathcal{R}, r, \rho, \mathfrak{d} \rangle$ whose components have the following types:

Object $\mathcal{R}$**:** **C**

Morphism r**:** $\mathcal{C} \to \mathcal{R}$

Action ρ**:** A mapping from each $\varepsilon : E$ to a 2-cell $\rho_\varepsilon : m_\varepsilon ; r \Rightarrow r$

Distributivity $\mathfrak{d}$**:** A proof that