Noninterference

Ross Tate

October 24, 2014

Definition (Power $\mathcal{E} \pitchfork \mathcal{C}$ for 2-Categories)**.** Suppose $\mathbf{C}$ is a 2-category. Suppose $\mathbf{E}$ is a category, and $\mathcal{C}$ in object of $\mathbf{C}$. Then a power of $\mathcal{C}$ by $\mathbf{E}$ is an object of $\mathbf{C}$, often denoted by $\mathbf{E} \pitchfork \mathcal{C}$, such that for any $\mathbf{C}$-object $\mathcal{D}$ there is an isomorphism between the categories $\mathbf{M_C}(\mathcal{D}, \mathbf{E} \pitchfork \mathcal{C})$ and $\mathbf{E} \multimap \mathbf{M_C}(\mathcal{D}, \mathcal{C})$ that is natural with respect to $\mathcal{D}$. Naturality here means that the two bifunctors from $[\mathbf{M_C}(\mathcal{D}_1, \mathcal{D}_2), \mathbf{M_C}(\mathcal{D}_2, \mathbf{E} \pitchfork \mathcal{C})]$ to $\mathbf{E} \multimap \mathbf{M_C}(\mathcal{D}_1, \mathcal{C})$ buildable from the isomorphisms and from concatonation as defined by $\mathbf{C}$ are equal.

Remark. Power is also known as cotensor.

Definition (Powered 2-Category)**.** A 2-category for which a power object $\mathbf{E} \pitchfork \mathcal{C}$ exists for all categories $\mathbf{E}$ and objects $\mathcal{C}$ of the 2-category.

Remark. The fact that there is an identity 1-cell from $\mathbf{M_C}(\mathbf{E} \pitchfork \mathcal{C}, \mathbf{E} \pitchfork \mathcal{C})$ implies that there is a functor $\pi : \mathbf{E} \to \mathbf{M_C}(\mathbf{E} \pitchfork \mathcal{C}, \mathcal{C})$. For every object $\mathcal{E}$ of $\mathbf{E}$, π maps $\mathcal{E}$ to some 1-cell from $\mathbf{E} \pitchfork \mathcal{C}$ to $\mathcal{C}$, and for every morphism $e : \mathcal{E}_1 \to \mathcal{E}_2$ of $\mathbf{E}$, π maps e to some 2-cell from $\pi_{\mathcal{E}_1}$ to $\pi_{\mathcal{E}_2}$. Furthermore, identity morphisms are mapped to identity 2-cells, and compositions of morphisms are mapped to compositions of 2-cells. This functor has the property that, for any object $\mathcal{D}$ of $\mathbf{C}$ with a functor $D : \mathbf{E} \to \mathbf{M_C}(\mathcal{D}, \mathcal{C})$, there exists a unique 1-cell $\langle D \rangle : \mathcal{D} \to \mathbf{E} \pitchfork \mathcal{C}$ such that for each object $\mathcal{E}$ of $\mathbf{E}$ the composition of 1-cells $\langle D \rangle \,;\pi_{\mathcal{E}}$ equals $D(\mathcal{E})$ and similarly equality of 2-cells holds for each morphism of $\mathbf{E}$. If there were another functor $D' : \mathbf{E} \to \mathbf{M_C}(\mathcal{D}, \mathcal{C})$ with a natural transformation $\alpha : D \Rightarrow D'$, then there would be a corresponding 2-cell $\langle \alpha \rangle : \langle D \rangle \Rightarrow \langle D' \rangle$.

Example. In $\mathbf{CAT}$, the power of $\mathbf{C}$ by $\mathbf{E}$, i.e. the object $\mathbf{E} \pitchfork \mathbf{C}$, is $\mathbf{E} \multimap \mathbf{C}$.

Theorem. *For any 2-category $\mathbf{C}$, the operation $\mathbf{E} \pitchfork \bullet$, if defined on all objects of $\mathbf{C}$, can be extended to a 2-endofunctor on $\mathbf{C}$.*