**Definition.** Given an object $C$ and subobjects $m_1 : S_1 \hookrightarrow C$ and $m_2 : \mathcal{S}_2 \hookrightarrow C$, define $m_1 \sqsubseteq_C m_2$ to be $\exists f : S_1 \to \mathcal{S}_2. m_1 = f \cdot m_2$.

**Theorem.** $\sqsubseteq_C$ is a preorder on the subobjects of $C$.

**Definition.** Given an object $C$ of a topos, define $\text{true}_C : C \to \Omega$ to be $\langle \rangle_C \cdot \text{true}$.

**Exercise 1.** Given an object $C$ of a topos and subobjects $m_1 : S_1 \hookrightarrow C$ and $m_2 : \mathcal{S}_2 \hookrightarrow C$, prove that $m_1 \sqsubseteq_C m_2$ holds if and only if $m_1 : \chi_{m_1} equals \text{true}_{S_1}$.

**Definition.** Given an object $C$ and a morphism $p : C \to \Omega$, let $m_p : S_p \hookrightarrow C$ be the (unique up to isomorphism) subobject produced by the pullback of $\text{true}$ and $p$.

**Exercise 2.** Given an object $C$ of a topos and subobjects $m_1 : S_1 \hookrightarrow C$ and $m_2 : \mathcal{S}_2 \hookrightarrow C$, let $p : C \to \Omega$ be defined as $\langle \chi_{m_1}, \chi_{m_2} \rangle \cdot \land$. Prove that $m_p$ is the meet of $m_1$ and $m_2$ with respect to the preorder $C$. Hint: take advantage of the following theorem.

**Theorem.** Given any commuting diagram of the following form (minus the dashed line), if the outer $[A, B, E, F]$ is a pullback square and the lower $[C, D, E, F]$ is a pullback square, then the upper $[A, B, C, D]$ using the uniquely induced dashed line is also a pullback square:

$$
\begin{array}{ccc}
A & \rightarrow & B \\
\downarrow & & \downarrow \\
C & \rightarrow & D \\
\downarrow & & \downarrow \\
E & \rightarrow & F \\
\end{array}
$$