

Factorizations

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Exercise 1. Prove that the category of ranked preorder semigroups has a particular $(\mathcal{E}, \mathcal{M})$ -factorization structure. A ranked preordered semigroup is a preordered semigroup $\langle M, \leq, * \rangle$ along with a function $r : M \rightarrow \mathbb{R}$ such that $r(m_1 * m_2) = r(m_1) + r(m_2)$, and $m \leq m'$ implies $r(m) \leq r(m')$. A morphism of ranked preorder semigroups $f : \mathcal{M}_1 \rightarrow \mathcal{M}_2$ is a morphism of preordered semigroups with the additional property that $r_2(f(m_1)) = r_1(m_1)$.

This category has an underlying functor to **Prost**, which we use to define the particular \mathcal{E} and \mathcal{M} we want a factorization structure for. A morphism belongs to \mathcal{E} iff its underlying morphism is an epimorphism in **Prost**. A source belongs to \mathcal{M} iff its underlying source is an initial monosource in **Prost**.

Show how to construct the category of ranked preorder semigroups from **Prost** using the techniques from class. I am happy if you give the correct construction, even if you do not do the detailed work of proving the result is isomorphic to the category of ranked preorder semigroups.

Remark. I made up the term *ranked*.