Exercise 1. Prove that the inclusion functor \( \textbf{Set} \xhookrightarrow{I} \textbf{Rel} \) has a right adjoint. You may use any of the equivalent definitions of adjunction. For clarification, \( I \) is the functor mapping each set \( X \) (an object of \( \textbf{Set} \)) to the set \( X \) (also an object of \( \textbf{Rel} \)) and each function \( X \rightarrow Y \) (a morphism of \( \textbf{Set} \)) to the relation \( \lambda(x, y). f(x) = y \) (a morphism of \( \textbf{Rel} \)).

Exercise 2. There is a functor from \( \mathbf{1} \) to \( \textbf{Set} \) picking out the empty set, and another functor from \( \mathbf{1} \) to \( \textbf{Set} \) picking out the singleton set. One is the left adjoint to the unique functor from \( \textbf{Set} \) to \( \mathbf{1} \), and the other is the right adjoint to the unique functor from \( \textbf{Set} \) to \( \mathbf{1} \). Determine and prove which is which.

Exercise 3. \( N : \mathbf{1} \rightarrow \textbf{Set} \) maps the only object of \( \mathbf{1} \) to the set \( \mathbb{N} \). \texttt{repeat} is the natural transformation from \( N \) to \( N ; L \) (i.e. \( L(N) \)) mapping the sole object of \( \mathbf{1} \) to the function mapping \( n \) to the length-\( n \) list \([n, \ldots, n]\). \texttt{sum} is the natural transformation from \( N ; L \to N \) mapping the sole object of \( \mathbf{1} \) to the function mapping a list of numbers and returns its sum.

The string diagram to the right denotes a natural transformation from the functor \( N : \mathbf{1} \rightarrow \textbf{Set} \) to itself (\( N \) maps the only object of \( \mathbf{1} \) to the set \( \mathbb{N} \)). In particular, this means it describes a function from \( \mathbb{N} \) to \( \mathbb{N} \). Determine what that function is in terms of basic arithmetic. (No proof necessary; the purpose of this is to learn the notation.)