1 Review of CCS

$$P ::= A\langle \overrightarrow{a} \rangle \mid \sum_{i \in \mathcal{I}} \alpha_i . P_i \mid P_1 | P_2 \mid \textsf{new}\ a\ P$$

2 Alegebraic Properties of Bisimilarity

An example of of the kinds of properties exhibited by bisimilarity is $a|b \sim a.b + b.a$

Proposition 1. $\forall P.\ P \sim \sum\{\alpha.P' \mid P \xrightarrow{\alpha} P'\}$

The summation is well-formed, since P' can contain only a finite number of parallel compositions and sums over finite domains.

Proposition 2.

$$\begin{array}{rcl} \multicolumn{3}{l}{\forall P_1 \dots P_n.} \\ P_1 \mid \dots \mid P_n & \sim & \sum\{\alpha.(P_1 \mid \dots \mid P_i' \mid \dots \mid P_n) \mid P_i \xrightarrow{\alpha} P_i' \wedge 1 \le i \le n\} \\ & + & \sum\{\tau.(P_1 \mid \dots \mid P_i' \mid \dots \mid P_j' \mid \dots \mid P_n) \\ & & \quad \mid P_i \xrightarrow{\lambda} P_i' \wedge P_j \xrightarrow{\bar{\lambda}} P_j' \wedge 1 \le i < j \le n\} \end{array}$$

Proposition 3 (Expansion Law)**.**

$$\begin{array}{rcl} \multicolumn{3}{l}{\forall P_1 \dots P_n, \overrightarrow{a}.} \\ \textsf{new}\ \overrightarrow{a}\ P_1 \mid \dots \mid P_n & \sim & \sum\{\alpha.\textsf{new}\ \overrightarrow{a}\ (P_1 \mid \dots \mid P_i' \mid \dots \mid P_n) \\ & & \quad \mid P_i \xrightarrow{\alpha} P_i' \wedge 1 \le i \le n \wedge \alpha, \bar{\alpha} \notin \overrightarrow{a}\} \\ & + & \sum\{\tau.\textsf{new}\ \overrightarrow{a}\ (P_1 \mid \dots \mid P_i' \mid \dots \mid P_j' \mid \dots \mid P_n) \\ & & \quad \mid P_i \xrightarrow{\lambda} P_i' \wedge P_j \xrightarrow{\bar{\lambda}} P_j' \wedge 1 \le i < j \le n \wedge \lambda, \bar{\lambda} \notin \overrightarrow{a}\} \end{array}$$

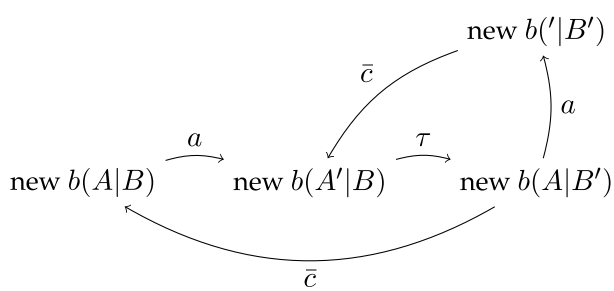
2.1 Expansion Law at Work

Consider the following process.

$$A = a.A' \qquad b = b.B'$$

$$A' = \bar{b}.A \qquad B' = \bar{c}.B$$

$$\textsf{new}\ b\ (A \mid B)$$



By the expansion law, the following properties hold for the above system.

$$\begin{array}{lcl} \mathsf{new}\ b\ (A \mid B) & \sim & a.\mathsf{new}\ b\ (A' \mid B) \\ \mathsf{new}\ b\ (A' \mid B) & \sim & \tau.\mathsf{new}\ b\ (A \mid B') \\ \mathsf{new}\ b\ (A \mid B') & \sim & a.\mathsf{new}\ b\ (A' \mid B') + \bar{c}.\mathsf{new}\ b\ (A' \mid B) \\ \mathsf{new}\ b\ (A' \mid B') & \sim & \bar{c}.\mathsf{new}\ b\ (A' \mid B) \end{array}$$

3 Sequential Composition as a Derived Form

We can express sequential composition in CCS as

$$P; Q = \mathsf{new}\ start\ (P[start/done] \mid start.Q)$$

where P signals $\overline{done}$ when it finishes.

Exercise

Show that $(P; Q); R \sim P; (Q; R)$

3.1 Congruence

An **elementary context** is a expression with one hole at the top-level. For instance, the elementary contexts for processes are

$$E ::= \sum_{i \in \mathcal{I}} \alpha_i.P_i + \alpha.\Box + \sum_{j \in \mathcal{J}} \alpha_j.P_j \ \mid\ \Box|P \ \mid\ P|\Box \ \mid\ \mathsf{new}\ a\ \Box$$

A **congruence** is a relation closed under substitution in related contexts.
C is a congruence $\iff$ $P\ C\ Q$ implies $E[P]\ C\ E[Q]$ for all processes P,Q and elementary contexts E.

Proposition 4 ($\sim$ is a congruence)**.**

If $P \sim Q$ then

- $\alpha.P + M \sim \alpha.Q + M$
- $\mathsf{new}\ a\ P \sim \mathsf{new}\ a\ Q$
- $P|R \sim Q|R$
- $R|P \sim R|Q$

4 Experiment LTS

Definition 1. An **experiment** is a sequence $\lambda_1, \ldots, \lambda_n \in \mathcal{L}^*$.

Based on the labeled transition system ($\rightarrow$) we defined for CCS, we now define an **experiment LTS**, whose transitions will be denoted by $\Rightarrow$.

- $P \Rightarrow Q \overset{\Delta}{\iff} \underbrace{P \rightarrow \ldots \rightarrow Q}_{\text{0 or more steps}}$

- $P \stackrel{e}{\Rightarrow} Q \stackrel{\Delta}{\iff} P \Rightarrow \stackrel{\alpha_1}{\rightarrow} \stackrel{\alpha_2}{\rightarrow} \ldots \stackrel{\alpha_n}{\rightarrow} P_n \Rightarrow Q$
  where $e = \alpha_1 \ldots \alpha_n$

We can intuitively interpret each type of transition as follows

$\Rightarrow$	0 or more reactions
$\stackrel{\tau}{\Rightarrow}$	1 or more reactions
$\stackrel{e}{\Rightarrow}$	Performing experiment e

Definition 2. A binary relation $\mathcal{S}$ is a **weak simulation** if and only if,
$(P\mathcal{S}Q \wedge P \stackrel{e}{\Rightarrow} P')$ implies $\exists Q'.\ (Q \stackrel{e}{\Rightarrow} Q' \wedge P'\mathcal{S}Q')$.

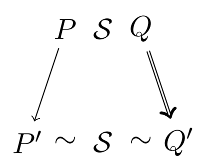
We say Q weakly simulates P if there exists a weak simulation $\mathcal{S}$ such that $P\mathcal{S}Q$.

Definition 3 (Weak Bisimulation)**.**
A binary relation $\mathcal{S}$ is a weak bisimulation if both $\mathcal{S}$ and its converse $\mathcal{S}^{-1}$ are weak simulations.

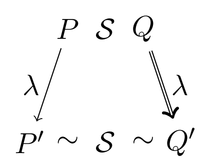
Definition 4 (Weak Bisimilarity)**.**
P is weakly bisimilar to Q, written $P \approx Q$, if there exists a weak bisimulation $\mathcal{S}$ such that $P\mathcal{S}Q$.

Proposition 5. $\mathcal{S}$ is a weak bisimulation if and only if the following hold whenever $P\mathcal{S}Q$.

1. If $P \rightarrow P'$ then $\exists Q'.\ Q \Rightarrow Q' \wedge P'\mathcal{S}Q'$
2. If $P \stackrel{\lambda}{\rightarrow} P'$ then $\exists Q'.\ Q \stackrel{\lambda}{\Rightarrow} Q' \wedge P'\mathcal{S}Q'$



(1)



(2)

Proposition 6.

1. $\approx$ is an equivalence relation.
2. $\approx$ is a weak bisimulation. (In fact, it is the largest one).
3. Every strong bisimulation is a weak bisimulation.
4. Similar to the definition of strong bisimulation up to congruence, we can define weak bisimulation up to strong bisimulation.

Proposition 7. The following subset relation holds: $= \subseteq \equiv \subseteq \sim \subseteq \approx$

Proposition 8. If $\mathcal{S}$ is a weak bisimulation up to strong bisimulation and $P\mathcal{S}Q$, then $P \approx Q$