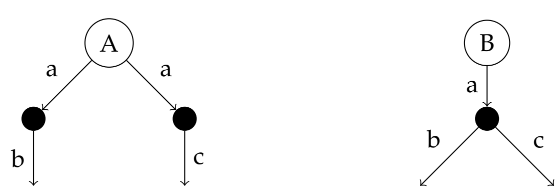
1 Review

- $\mathcal{L} = \aleph \biguplus \bar{\aleph}$. Partition the set of actions into two disjoint sets. Note that actions $a, \bar{a}$ are complementary actions.
- $Act = \mathcal{L} \bigcup \{\tau\}$, where Act is the action-set.
- $P ::= A\langle\vec{a}\rangle | \sum_{i \in I} \alpha_i.P_i | P_1 | P_2 | \text{new}\alpha P$
- Recall the rules necessary for structural congruence
  - Sum: $\dfrac{}{M + \alpha.P + N \xrightarrow{\alpha} P}$
  - React: $\dfrac{P \xrightarrow{\lambda} P', Q \xrightarrow{\bar{\lambda}} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$
  - Par-L: $\dfrac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q}$
  - Par-R: $\dfrac{Q \xrightarrow{\alpha} Q'}{P|Q \xrightarrow{\alpha} P|Q'}$
  - Res: $\dfrac{P \xrightarrow{\alpha} P'}{\text{new}\alpha P \xrightarrow{\alpha} \text{new}\alpha P'}$ if $\alpha \notin \{a, \bar{a}\}$
  - Ident: $\dfrac{\{\frac{\vec{b}}{\vec{a}}\}P_A \xrightarrow{\alpha} P'}{A\langle\vec{b}\rangle \xrightarrow{\alpha} P'}$ if $A(\vec{a}) \stackrel{\text{def}}{=} P_A$

2 Notion of Bisimulation

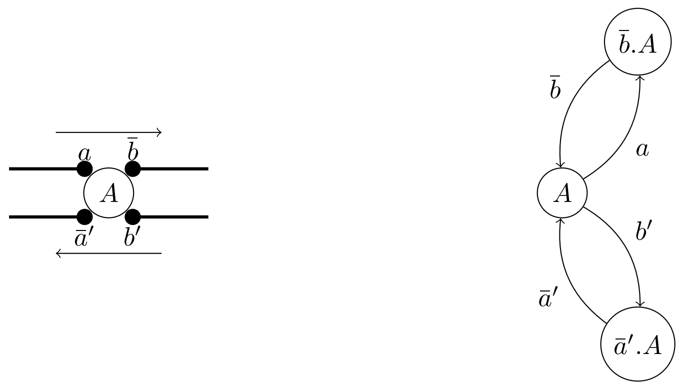


The language of A: $\mathcal{L}(A) = \{\text{ab, ac}\}$. The language of B: $\mathcal{L}(B) = \{\text{ab, ac}\}$. Both processes recognize the same language. However, as can be seen, B simulates A, but A does not simulate B. Therefore, language equivalence is not good enough.

2.1 Example: One-place buffer

Let $P(x, x', y, y') = x.\bar{y}.P\langle x, x', y, y'\rangle + y'.\bar{x}'.P\langle x, x', y, y'\rangle$
Let A be an instance of P:

$$\begin{aligned} A &= P\langle a, a', b, b'\rangle \\ &\equiv a.\bar{b}.A + b'.\bar{a}'.A \end{aligned}$$



Act, $\overline{Act}$ are complementary; by convention, $\overline{Act}$ is designated as output

Figure 2.1: Structural diagram (left) and labeled transition system (right) of a one-place buffer

2.2 Example: Two-place buffer

Define another process B, also an instance of P:

$$\begin{aligned} B &= P\langle b, b', c, c'\rangle \\ &\equiv b.\bar{c}.B + c'.\bar{b}'.B \end{aligned}$$

A Two-place buffer is the parallel composition of processes A and B, $(A|B)$

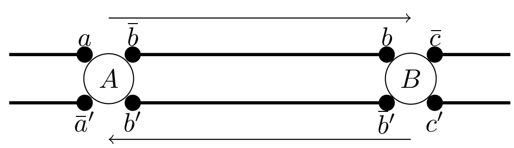


Figure 2.2: Structural Diagram for two-place buffer

2.3 Example: Semaphores

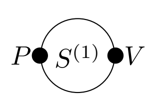


Figure 2.4: Structural Diagram for a one-place semaphore

Consider the definitions below for one-ary (left) and two-ary (right) semaphores:

$$\begin{aligned} S^{(1)} &= p.S_1^{(1)} \\ S_1^{(1)} &= v.S^{(1)} \end{aligned} \qquad \begin{aligned} S^{(2)} &= p.S_1^{(2)} \\ S_1^{(2)} &= p.S_2^{(2)} + v.S^{(2)} \\ S_2^{(2)} &= v.S_1^{(2)} \end{aligned}$$

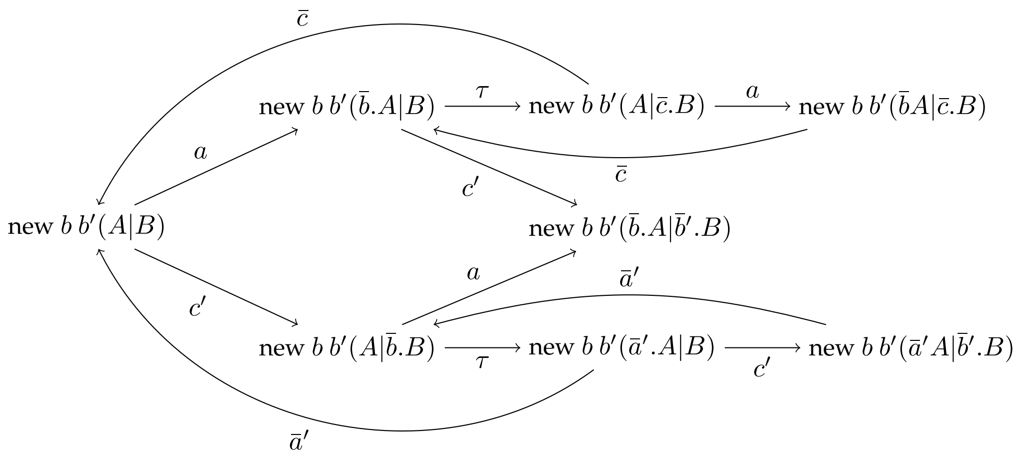


Figure 2.3: Labeled Transition System for a two-place buffer

Proposition. $S^{(1)}|S^{(1)} \sim S^{(2)}$

Proof. Proving this is equivalent to showing that:

$$\begin{aligned}\mathcal{R} = \{&(S^{(1)}|S^{(1)}, S^{(2)}), (S_1^{(1)}|S^{(1)}, S_1^{(2)}),\\ &(S^{(1)}|S_1^{(1)}, S_1^{(2)}), (S_1^{(1)}|S_1^{(1)}, S_2^{(2)})\}\end{aligned}$$

This is a strong bisimulation, since, for every transition in $S^{(1)}|S^{(1)}$, there is a transition in $S^{(2)}$. Therefore, $S^{(1)}|S^{(1)}$ and $S^{(2)}$ are bisimilar $(\sim)$ □

3 Strong Simulation

Definition 1. [Strong Simulation up to $\equiv$ (structuralcongruence)]
A binary relation S is a strong simulation up to $\equiv$ if, whenever PSQ, if $P \xrightarrow{\alpha} P'$ then $\exists Q'$ such that $Q \xrightarrow{\alpha} Q'$ and $P' \equiv S \equiv Q'$.

Note: $P' \equiv S \equiv Q'$ is a relational composition

$$\begin{array}{ccc} P & S & Q \\ \alpha \swarrow & & \searrow \alpha \\ P' & \equiv S \equiv & Q' \end{array}$$

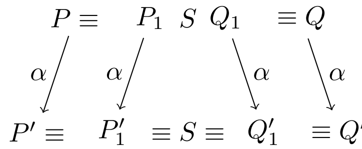
Figure 3.1: Criteria for Strong Simulation

Proposition. *If S is a strong bisimulation of up to $\equiv$ and PSQ, then $P \sim Q$*

Proof. We must show that $\equiv S \equiv$ is a strong bisimulation. If this is true, then $P \sim Q$. Let $P \equiv S \equiv Q$ and $P \xrightarrow{\alpha} P'$. We want to find a Q' that completes the following:

$$\begin{array}{ccc} P & \equiv S \equiv & Q \\ \alpha \downarrow & & \downarrow \alpha \\ P' & \equiv S \equiv & Q' \end{array}$$

Note that for some P_1 and Q_1, $P \equiv P_1$, $P_1 S Q_1$, and $Q_1 \equiv Q$. Given the transitivity of $\equiv$,

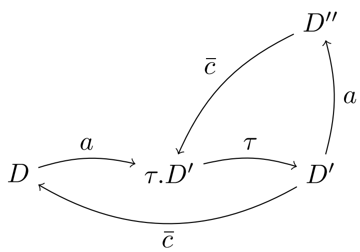


Given that structural congruence is a strong bisimulation, and the transitivity of $\equiv$, we have $P \sim Q$.

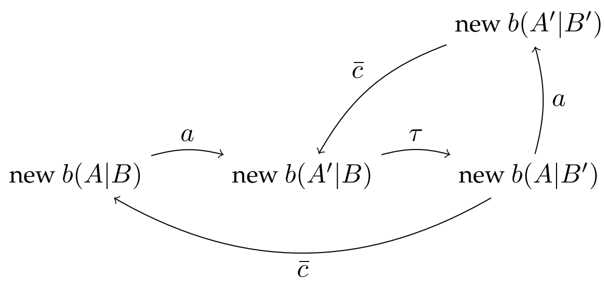
□

3.1 Another Example

$$D = a.\tau.D'$$
$$D' = a.D'' + \bar{c}.D$$
$$D'' = \bar{c}.\tau.D'$$



$$A = a.A'$$
$$A' = \bar{b}.A$$
$$B = b.B'$$
$$B' = \bar{c}.B$$



These two processes bisimulate each other. If the rules for the left-hand process were changed to $D = a.\tau.\tau.D'$ and $D'' = \bar{c}.\tau.\tau.D'$, then they are no longer a strong bisimulation. However, they are still weakly bisimilar.

4 Algebraic Properties

- $a|b \sim a.b + b.a$
- $\forall P, P \sim \sum\{\alpha.P' | P \xrightarrow{\alpha} P'\}$