

face the problem of the consistency of Arithmetic, Hilbert sought a solution based on this very distinction. In his paper *Neubegründung der Mathematik* he launched the term 'meta-mathematics', 'the contentual theory of formalized proofs', a theory or system which, as analysis of mathematical theory, deals with formulae and formula-construction and is finite in character. Brouwer's observation that the mathematical structure of such analysis was a simple and basic one was used by Hilbert to seek the justification of mathematics in the finiteness of his meta-mathematics and turn it into the missing link in the somewhat circular chain of his consistency proofs.

By the end of 1927 Brouwer's fortunes seemed to be at their lowest ebb. His programme of re-constructing mathematics had run aground. His dreams of a Mathematics Institute in Amsterdam, the recognized centre of the mathematical world, were shattered. Almost all international support had fallen away. Hilbert's renewed and public attack on Brouwer and, even more, his annexation of meta-mathematics 'without mention of autorship', turning it into a weapon against Intuitionism, was the last straw, returning the attack in an emotional outburst, *Intuitionistische Betrachtung über den Formalismus* [1928A], Brouwer first proved from his own and from Hilbert's work his paternity claim of the notion 'meta-mathematics' and then listed the improvements in the Formalist Programme due to his questioning of the validity of the Principle of the Excluded Middle:

Formalism has received nothing but benefits from Intuitionism and can expect further benefits. The Formalist School should therefore show due recognition instead of war-mongering against Intuitionism in sneering tones, never once making proper reference to autorship. Moreover, Formalism should remember that in the Formalist structure so far nothing mathematical has been achieved (we are still waiting for a proof of the non-contradictoriness of its axiom system), whereas Intuitionism on the basis of its constructive definition of sets and the Fundamental Property of Finite Sets has already erected new structures in real mathematics of unshakable certainty. ([1928A], p.4).

\* In his letters to Hilbert of 28.10.09 and 28.08.18 Brouwer refers to Hilbert's and Mrs Hilbert's visit and their meeting in the Hague in the autumn of 1909, 'their walk in the dunes... and their exciting conversation'. In [1928A], footnote 2 Brouwer claims that on that occasion he spoke to Hilbert about his distinction between 'mathematics' and 'mathematics of second order'.

The last stages in the Brouwer-Hilbert battle were fought on a semi-political front and concerned the International Congress of Mathematicians at Bologna in September 1928.

The Bologna Congress was organized by the Union Mathématique Internationale under the auspices of the Conseil International de Recherches. Brouwer's public call in 1928 on all German mathematicians to boycott the Congress followed years of his campaigning for the admission of German mathematicians as full members of the Union and the removal from its constitution of the humiliating references to German scholarship (cf. also 2.9.3). Hilbert, however, interpreted Brouwer's action as interference in German affairs, a new obsession of Brouwer, disguising a deliberate attempt to prevent him from attending the Congress as head of the German delegation.

Hilbert's address to the Bologna Congress was his last public appearance before retirement; for a moment his leadership seemed undisputed and his Programme almost complete. Brouwer stayed away and so did many leading German mathematicians.

For Hilbert, however, the matter was not closed; it had shown up Brouwer's ambitions and personal influence. Hilbert felt confident of the victory of his Formalist programme and the loyalty of his Göttingen Circle, but he feared Brouwer's strong personality and his influence on the Editorial Board of the *Mathematische Annalen*, which he considered as almost his personal property. On his return from Bologna in a curt note he dismissed Brouwer from the Editorial Board, without even first obtaining the permission of Einstein and Carathéodory, who with Hilbert formed the chief editorial board. The only reason given was: 'the incompatibility of our views on Foundations, which makes it impossible for me to work with you' (Hilbert to Brouwer, 25.10.28).

Hilbert's friends and co-editors were taken by surprise, but they were anxious to avoid any unpleasantness. Schmidt sent a telegram to Brouwer; Carathéodory, one of Brouwer's few remaining friends and admirers, was dispatched to Laren to persuade Brouwer not to take immediate action, because 'Hilbert is desperately ill and will regret his steps in a few weeks'. Brouwer, however, soon found out that his co-editors' first and only concerns were the wishes of their master and that they were prepared to sacrifice Brouwer in the process. To him the dismissal from the Editorial Board was an injustice and a blow to his career, which he was not prepared to accept without a struggle. The frantic correspondence between Blumenthal, Courant, Carathéodory, Springer, Einstein, Bohr and Brouwer



gives a day-to-day account of the 'Annalenstreit' and the almost conspiratory efforts of the inner circle to legitimize Hilbert's rash action. When legal advice was sought it became clear that Hilbert had exceeded his authority, dismissal could only be effected by a unanimous decision of the chief editors; both Carathéodory and Einstein remained unwilling to sign the dismissal notice and a considerable number of assistant-editors were opposed to Brouwer's removal. Legal niceties had to be bent; the only solution was to disband the present company and start a new journal with Hilbert as the sole, supreme authority to appoint editors. Carathéodory and Einstein declined a seat on the new editorial board in spite of repeated pleas of Blumenthal and Courant. Carathéodory felt so strongly about this 'dishonourable affair' that he considered turning his back on Europe and accepting a chair at Stamford. Einstein, relieved at the end of the 'frog and mouse battle', refused the 'glamour of his name' on the cover page of the new Annalen.

In all these dealings Brouwer's continued membership of the editorial board was never at issue; Hilbert's decision, however unconstitutional was accepted by all except a few Berlin dissidents among board members. The debate among the senior board-members was about the justification of Brouwer's dismissal, no more than a post-mortem analysis of Hilbert's motives. Blumenthal in his circular letter to his fellow-editors had to admit that Brouwer's work as editor could not be faulted: 'Brouwer has been a very conscientious and capable editor' (Blumenthal to all editors of the MA 16.11.28). Carathéodory reacted angrily when Hilbert referred to the Bologna affair as the only reason for his decision: 'Brouwer has insulted me' (Carathéodory to Courant, 23.12.28). Blumenthal and Courant were more concerned about Hilbert's reputation and tried to persuade Carathéodory to accept and spread their agreed version:

Hilbert feared that Brouwer's personality might be damaging and dangerous for the future of the Annalen. It is not 'an interpretation construed after the event' if one emphasizes this factual motive, even if Hilbert's action at first might give a different impression. For Hilbert's sake we cannot allow a version of his reasons to become public which does not do him full justice. If you already accept such a version what can we expect from the public at large? (Courant to Carathéodory, 23.12.28).



Brouwer with his wife and Cor Jongejan, his secretary and life-long companion – 1927

## 2.14 The Silent Years

The Annalen Affair left Brouwer bitter and disillusioned and marks the end of his creative life. It came at the end of a growing crisis of confidence