So $\models \{A\}c\{B\}$ iff for all interpretations $I$, if $c$ is executed from a state which satisfies $A$ then if its execution terminates in a state that state will satisfy $B$.\(^1\)

**Exercise 6.7** In an earlier exercise it was asked to write down an assertion $A \in \text{Assn}$ with one free integer variable $i$ expressing that $i$ was prime. By working through the appropriate cases in the definition of the satisfaction relation $\models^I$ between states and assertions, trace out the argument that $\models^I A$ iff $I(i)$ is indeed a prime number. \(\square\)

### 6.4 Proof rules for partial correctness

We present proof rules which generate the valid partial correctness assertions. The proof rules are syntax-directed; the rules reduce proving a partial correctness assertion of a compound command to proving partial correctness assertions of its immediate subcommands. The proof rules are often called *Hoare rules* and the proof system, consisting of the collection of rules, *Hoare logic*.

**Rule for skip:**

$$\{A\} \text{skip} \{A\}$$

**Rule for assignments:**

$$\{B[a/X]\} X := a\{B\}$$

**Rule for sequencing:**

$$\{A\}c_0\{C\} \quad \{C\}c_1\{B\}
\{A\}c_0; c_1\{B\}$$

**Rule for conditionals:**

$$\{A \land b\}c_0\{B\} \quad \{A \land \neg b\}c_1\{B\}
\{A\} \text{if } b \text{ then } c_0 \text{ else } c_1\{B\}$$

**Rule for while loops:**

$$\{A \land b\}c\{A\}
\{A\} \text{while } b \text{ do } c\{A \land \neg b\}$$

**Rule of consequence:**

$$\models (A \Rightarrow A') \quad \{A'\}c\{B'\} \quad \models (B' \Rightarrow B)
\{A\}c\{B\}$$

\(^1\)The picture suggests, incorrectly, that the extensions of assertions $A'$ and $B'$ are disjoint; they will both always contain $\perp$, and perhaps have other states in common.