

Problem Set 1

Reading

Please read Thompson 2.1-2.5.

Exercises

An additional problem will be assigned by Friday January 30.

1. Solve exercises 2.2, 2.3, 2.4 on page 39 of Thompson. (Also attached to end of PS1)
2. Recall that λ -terms are α -equal iff they are the same except for the names of bound variables, e.g. $\lambda(x.x) =_{\alpha} \lambda(y.y)$. Prove the following result about substitution, for t, a, b λ -terms. Use the condition that x is not free in b .

$$t[a/x][b/y] =_{\alpha} t[b/y][a[b/y]/x]$$

Hint: Use induction on the structure of t .

3. Define lambda terms in the PL or PA you know best and give a capture avoiding substitution algorithm. (PL is Programming Language and PA is Proof Assistant-such as Agda, Coq, HOL, Nuprl).
4. Rewrite the capture example from Lecture 3 in two different ways:
 - (a) Using Barendregt's Variable Convention
 - (b) In Thompson's lambda calculus notation or in OCaml using integer expressions in place of $b(x, y)$ and $a(y)$.
5. Suppose you are a professor of Newtonian physics. You assigned the following problem as a homework:

“Suppose a car (say A) starts moving (from rest) in a straight line with a constant acceleration. Define a function f that expresses the distance covered in 10 seconds as a function of the acceleration (in m/s^2) of the car A. Now suppose that there is another car (say B) that starts moving from the same position with a constant acceleration of $2m/s^2$ in the opposite direction. Define a function g that expresses the distance between cars A and B at 10 seconds as a function of the speed of the car A”

A student submitted the following erroneous answer:

Speed of the car A at time x is the value of its acceleration multiplied by x . So,

$$f(y) = \int_0^{10} y \times x \, dx \quad (1)$$

In the frame of the car B, A's acceleration increases by $2m/s^2$. So,

$$g(x) = f(2 + x) \quad (2)$$

$$= \int_0^{10} (2 + x) \times x \, dx \quad (3)$$

$$= [x^2 + x^3/3]_0^{10} \quad (4)$$

$$= 100 + 1000/3 \quad (5)$$

How would you explain the mistake to them?