Problem Set 4

October 19, 2014

1. Write the inductive proof that $\forall x, y. \exists z. \text{Add}(x, y, z)$. This proof will define an addition function as realizer. Compare it to the primitive recursive function $\text{add}(x, y)$.

Your proof will also apply to $\forall x, y. \exists z. \text{add}(x, y) = z$. How does the realizer compare to this function?

2. Ed Nelson does not believe that primitive recursive definition of $\text{exp}(x, y)$ defines a legitimate function. He does agree that that $\text{add}$ and $\text{mult}$ as defined recursively are legitimate. Can you think of any reason for his skepticism?

3. We can define the cost of a primitive recursive function as the number of successor applications necessary to produce the final value. What is the cost of $\text{add}$, $\text{mult}$, $\text{exp}$?

4. Prove the principle of course of values induction with base from standard induction.

5. The Least Number Principle is this theorem of Peano Arithmetic. Is this valid constructively? Explain.

$\exists x. A(x) \Rightarrow \exists y. (A(y) \& \forall z. (z < y \Rightarrow \neg A(z)))$. 