

Suggested Exercises

We will grade these exercises and record them to help you learn the material and to count toward improving your grade and diagnosing your progress. To be graded we need these done by Tuesday Sept. 27.

1. Write ML functions that show that the following formulas in propositional logic are valid.

$$(a) (P \Rightarrow (Q \Rightarrow R)) \Rightarrow ((P \& Q) \Rightarrow R)$$

$$(b) (P \Rightarrow Q) \Rightarrow (Q \Rightarrow L) \Rightarrow (P \Rightarrow L)$$

(c) Explain how (b) is related to $(P \Rightarrow Q) \Rightarrow (\neg Q \Rightarrow \neg P)$

$$(d) (\neg(P \& Q) \& (P \vee \neg P) \& (Q \vee \neg Q)) \Rightarrow \neg P \vee \neg Q$$

2. Write the dependent types for these first-order formulas over a domain D .

$$(a) \forall x \exists y R(x, y)$$

$$(b) \forall x P(x) \Rightarrow \exists y Q(y)$$

$$(c) \forall x (P(x) \Rightarrow \exists y Q(x, y))$$

3. Write type theory or ML evidence for these formulas

$$(a) \forall x (P(x) \& Q(x)) \Rightarrow \forall x P(x) \& \forall x Q(x)$$

$$(b) (\exists x P(x) \& \forall x (P(x) \Rightarrow Q(x))) \Rightarrow \exists y Q(y)$$

4. Prove formula 1 (a) using Refinement Logic and extract the evidence term.

5. Make up an example of your own like problem 4.