Suggested Exercises

We will grade these exercises and record them to help you learn
the material and to count toward improving your grade and
diagnosing your progress. To be graded we need these done
by Tuesday Sept. 27.

1. Write ML functions that show that the following formulas
in propositional logic are valid.

(a) \((P \Rightarrow \neg Q \Rightarrow R) \Rightarrow ((P \& Q) \Rightarrow R)\)

(b) \((P \Rightarrow Q) \Rightarrow (Q \Rightarrow \bot) \Rightarrow (P \Rightarrow \bot)\)

(c) Explain how (b) is related to\n\((P \Rightarrow Q) \Rightarrow (\neg Q \Rightarrow \neg P)\)

(d) \((\neg (P \& Q) \& (P \lor \neg P) \& (Q \lor \neg Q)) \Rightarrow \neg P \lor \neg Q\)

2. Write the dependent types for these first-order
formulas over a domain \(D\).

(a) \(\forall x \exists y \ R(x, y)\)

(b) \(\forall x \ P(x) \Rightarrow \exists y \ Q(y)\)

(c) \(\forall x \ P(x) \Rightarrow \exists y \ Q(x, y)\)

3. Write type theory or ML evidence for these formulas

(a) \(\forall x \ (P(x) \& Q(x)) \Rightarrow \forall x \ P(x) \& \forall x \ Q(x)\)

(b) \(\exists x \ P(x) \& \forall x \ (P(x) \Rightarrow Q(x)) \Rightarrow \exists y \ Q(y)\)

4. Prove formula (a) using refinement logic and extract
the evidence term.

5. Make up an example of your own like problem 4.