CS 5860 Thur Sept 15, 2011

Lecture 6

PLAN

1. Discuss mini-projects
2. Note readings
3. Notation
4. Tableau proofs
5. Historical perspective on Computational Logic
6. Examples of computational content of propositions

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4. Tableau Proofs – examples using Smullyan rules p. 17 (also see p. 22)

The idea is to search for a counter example

1. $F(X \supset Y)$	vs	1. $F(X \supset X)$	α rule
2. TX		2. TX	α_1
3. FY		3. FX	α_2
counter example open tableau		$\times$ closed tableau	

1. $F(X \supset (Y \supset X))$
2. TX
3. $F(Y \supset X)$
4. TY
5. FX

$\times$ 2,5

1. $F((X \& (X \supset Y)) \supset Y)$
2. $T(X \& (X \supset Y))$
3. FY
4. TX from 2.
5. $T(X \supset Y)$ from 2 β-rule

6. FX	7. TY
$\times$ 4,6	$\times$ 3,7

5. Historical perspective

Euclid's Elements of Geometry is mostly constructive.

for an object to exist, it must be constructed

by ruler and compass, e.g. regular triangle, regular pentagon.

Note, on March 30, 1796 Gauss (Carl ~~Fred~~ Friedrich) reported constructing a regular 17gon, solving a 2,000 year old geometry problem and starting a diary ended only on July 9, 1814, after 146 mathematical results.

When we say $\exists x. P(x)$ the question is always, can we construct it. Consider this example.

$$\exists x, y : \text{Irrational}. (x^y \text{ is rational})$$

Here is a "non-constructive" proof.

Consider $\sqrt{2}^{\sqrt{2}}$, either this is rational or not, that is

$$\text{Rational}(\sqrt{2}^{\sqrt{2}}) \vee \sim \text{Rational}(\sqrt{2}^{\sqrt{2}})$$

If it is rational, then take $x = \sqrt{2}$ and $y = \sqrt{2}$.

If it is not rational, then it is irrational, so take $x = \sqrt{2}^{\sqrt{2}}$ and $y = \sqrt{2}$.

Then $(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = \sqrt{2}^2 = 2$ as required. Qed.

But what are the first seven digits of x?

are they 1.414213 or 1.189207 ?

We don't know because we haven't constructed a ~~any~~ unique x.

We see here that it is our understanding of or which is the problem.

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Lecture 6 continued

6. Examples of computational content of propositions.

1. The meaning of (evidence for) $X \Rightarrow X$ is a computable function taking evidence for X, say x, into evidence for X. Clearly the identity function works.

   ML type $'X \to 'X$

   ML notation: $\backslash x.x$

   Nuprl notation: $\lambda(x.x)$

2. Here is evidence for $X \Rightarrow (Y \Rightarrow X)$

   ML type $'X \to ('Y \to 'X)$

   ML notation: $\backslash x.(\backslash y.x)$ for evidence, a program

   Nuprl notation: $\lambda(x.\lambda(y.x))$

3. Here is evidence for $(A \& (B \vee C)) \Rightarrow ((A \& B) \vee (A \& C))$

   ML notation for evidence:

   $\backslash h$ let $h = \langle a, bc\rangle$ in
   if isl(bc) then inl(a, outl(bc))
   else ~~ou~~ inr(a, outr(bc))

   Nuprl notation:

   $\lambda(h.$ spread$(h; a, bc.$
   decide$(bc;\ b.$ inl$\langle a,b\rangle;\ c.$ inr$\langle a,c\rangle)))$

Exercise. Let $\perp$ be a constant (any in ML), find evidence for $((A \vee (A \Rightarrow \perp)) \Rightarrow \perp) \Rightarrow \perp$. The ML type is $(('A + 'A \to any) \to any) \to any$.