CS 5860

Thur Oct 13, 2011 Formalizing Arithmetic continued

PLAN

1. Review discussion of first-order expression of arithmetic expressing zero, successor, addition, multiplication

2. Axioms for equality (Eq), zero (Z), successor (Succ)

3 Discussion of projects involving FOL

4. Return homework

Axioms for equality and their evidence

$\forall x. Eq(x,x)$ reflexivity

We take the evidence for this to be $*$, a term showing that $Eq(x,x)$ is not empty.

$\forall x,y,z. (Eq(x,y) \& Eq(y,z) \Rightarrow Eq(x,z))$ transitivity

The evidence is $\lambda(x,y,z. \lambda(p. spread(p; l,r. *)))$; that is as long as there is l evidence for $Eq(x,y)$ and r for $Eq(y,z)$, then $*$ will be evidence for $Eq(x,z)$

$\forall x,y. (Eq(x,y) \Rightarrow Eq(y,x))$ symmetry

The evidence is $\lambda(x,y. \lambda(e.*))$ or even $\lambda(x,y. \lambda(z.z))$ which takes evidence such as $*$ for $E(x,y)$ and provides evidence for $Eq(y,x)$.

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Axioms about Zero

- $\exists x. Zero(x)$ there is a zero element of D, the evidence is $\langle 0, * \rangle$ or we could write $\langle d_0, * \rangle$. The element, say 0, must be in the domain D.

$\forall x, y. (Zero(x) \& Zero(y) \Rightarrow Eq(x,y))$, the zero element is unique.

The evidence is $\lambda(x, y. \lambda(p. *))$.

Axioms about Successor

$\forall x. \exists y. Succ(x,y)$ for every x there is a successor

The evidence is $\lambda(x. \langle s(x), * \rangle)$, so $*$ shows that we know axiomatically $Succ(x, s(x))$ is true or that $s(x)$ is the successor of x, and $s(x)$ belongs to D.

We can now be sure that

$0, S(0), S(S(0)), S(S(S(0))), \ldots$ belong to D.

We know these by their "decimal names"

$0, 1, 2, 3, \ldots$