CS 5860 Homework 1

Norris Xu (nx26)

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1. (a) \f.\x.**let** (p, q) = x **in** f p q

   (b) \f.\g.\p.g (f p)

   (c) In computational logic, we define negation ($\neg P$) by creating a new constant with no evidence, false (or $\bot$), and defining $\neg P \equiv P \to \bot$. Then (b) is simply $(P \Rightarrow Q) \Rightarrow (\neg Q \Rightarrow \neg P)$ expressed in computational logic using this definition.

   (d)
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   \x.let (f, a, b) = x in
       if !isl(a) then
           inl(outr(a))
       else if !isl(b) then
           inr(outr(b))
       else
           inl(\p.f (outl(a), outl(b)))
   ```

2. (a) $x : D \to (y : D \times R(x, y))$

   (b) $(x : D \to P(x)) \to (y : D \times Q(y))$

   (c) $x : D \to (P(x) \to (y : D \times Q(x, y)))$

3. (a) $(x : D \to (P(x) \times Q(x))) \to ((x : D \to P(x)) \times (x : D \to Q(x)))$
   $\lambda f.\mathsf{pair}(\lambda x.\mathsf{first}(f\ x), \lambda x.\mathsf{second}(f\ x))$

   (b) $((x : D \times P(x)) \times (x : D \to (P(x) \to Q(x)))) \to (y : D \times Q(y))$
   $\lambda a.\mathsf{spread}(a; b, f.\mathsf{spread}(b; x, p.\mathsf{pair}(x, f\ x\ p)))$

4.

$\vdash (P \Rightarrow (Q \Rightarrow R)) \Rightarrow ((P \wedge Q) \Rightarrow R)$	$\lambda f.[\_]$
$f : (P \Rightarrow (Q \Rightarrow R)) \vdash (P \wedge Q) \Rightarrow R$	$\lambda p.[\_]$
$f : (P \Rightarrow (Q \Rightarrow R)), p : (P \wedge Q) \vdash R$	$\mathsf{spread}(p; l, r.[\_])$
$f : (P \Rightarrow (Q \Rightarrow R)), l : P, r : Q \vdash R$	$\mathsf{ap}(f; [\_]; g.[\_])$
$l : P \vdash P$	l
$g : Q \Rightarrow R, r : Q \vdash R$	$\mathsf{ap}(g; [\_]; h.[\_])$
$r : Q \vdash Q$	r
$h : R \vdash R$	h

From this, we can extract the evidence term:

$$\lambda f.\lambda p.\mathsf{spread}(p; l, r.\mathsf{ap}(f; l; g.\mathsf{ap}(g; r; h.h)))$$

5. Formula 1(b):

$$\begin{array}{rl}
\vdash (P \Rightarrow Q) \Rightarrow ((Q \Rightarrow \bot) \Rightarrow (P \Rightarrow \bot)) & \lambda f.[_] \\
f : (P \Rightarrow Q) \vdash (Q \Rightarrow \bot) \Rightarrow (P \Rightarrow \bot) & \lambda g.[_] \\
f : (P \Rightarrow Q), g : Q \Rightarrow \bot \vdash P \Rightarrow \bot & \lambda p.[_] \\
f : (P \Rightarrow Q), g : Q \Rightarrow \bot, p : P \vdash \bot & \mathsf{ap}(f; [_]; q.[_]) \\
p : P \vdash P & p \\
g : Q \Rightarrow \bot, q : Q \vdash \bot & \mathsf{ap}(g; [_]; b.[_]) \\
q : Q \vdash Q & q \\
b : \bot \vdash \bot & b
\end{array}$$

From this, we can extract the evidence term:

$$\lambda f.\lambda g.\lambda p.\mathsf{ap}(f; p; q.\mathsf{ap}(g; q; b.b))$$