CS 5860 Final Homework Assignment

No one managed a correct inductive proof of problem 2 from the oct 25 assignment, to show that equality on natural numbers is decidable, i.e.

$$\forall x. \forall y. ((x = y) \vee \sim(x = y))$$

This can be proved by induction on x first. In the base case we use a simple axiom. In the induction case, to show $\forall y\, ((x = y) \vee \sim(x = y)) \vdash \forall y\, ((x+1 = y) \vee \sim(x+1 = y))$

we use induction on y. We proceed as done in class on Nov 22, using Dr. Rahli's suggestion for how to use the nested induction hypotheses. With his approach the proof is easy. Attach your solution to your project, Due Wed. December 7.

1. Prove the result by induction as suggested above.
2. Write out the realizer for induction and explain intuitively how equality is decided.

Extra Credit

Define	$x < y$	iff	$\exists z. (x + (z+1) = y)$
" "	$x \leq y$	iff	$x < y \vee x = y$
	$x > y$	iff	$\exists z. (y + (z+1) = x)$
	$x \geq y$	iff	$x > y \vee x = y$

Show

$$\forall x\, \forall y. ((x < y) \vee (x = y) \vee (x > y))$$

This law is called Trichotomy.